INTRODUCTORY MATHEMATICAL ECONOMICS

Questions	168
Field	Economics
Target Audience	Economics and Commerce
Target Level	Second-year Undergraduate

Topics

- Rules of Differentiation
- First Order Differential Equations
- Higher Order Derivatives
- Optimization in One Variable
- Second Order Conditions for Optimization
- Systems of Linear Equations
- Optimization with Direct Restrictions on Variables
- Over Determined and Under Determined Systems
- Matrix Representation of Systems
- Gauss Jordan
- Matrix Operations
- Types of Matrices
- Determinants and Inverses
- Partial Differentiation
- Second Order Partial Derivatives
- Multivariate Optimization
- Second Order Conditions for Multivariate Optimization
- Multivariate Optimization with Direct Restrictions of Variables
- Constrained Optimization and the Lagrangean Method
- Second Order Conditions for Constrained Optimization

Outline

The material in this module is designed to cover a single-semester course in mathematical economics for economics and commerce students at the second-year university level. The questions are designed to span the topics listed above, allowing for practice, homework or testing throughout the semester.

The vast majority of the questions are algorithmic and take numeric or algebraic responses. Dynamically labelled diagrams are included where appropriate. Information fields are included on all questions indicating topics and difficulty level.

The module was first implemented in Winter 2012 at the University of Guelph and subsequently used in Summer 2012 for the distance education version of the course, and has seen a several rounds of updates.

For the on campus format, the questions were used in weekly assignments that were open for unlimited practice in the first half of the week, and in the second half the graded version was opened and students were given the best out of five attempts. In the distance format, there were fewer assignments with more questions and with more time given for both the practice portion and for the graded version.

Sample homework assignments are provided, spanning the course material. The included Maple TA Syntax Sheet was provided to the students in the course offerings and covers many common student response situations.

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MAPLE TA SYNTAX SHEET

Expression	Entry Syntax
$x \cdot y$	x*y
$\frac{x}{y}$	x/y
x^y	x^y
$\frac{a}{b \cdot c}$	a/(b*c) (although it will accept a/b/c)
$\sqrt{x}$	sqrt(x) or x^(1/2) (do not use x^0.5)
$x^{\frac{2}{3}} = \sqrt[3]{x}$	x^(2/3)
$\lvert x \rvert$	abs(x)
$\ln(x)$	ln(x)
$\log_n(x)$	log[n](x)
e^x	exp(x)
e	e or exp(1)
π	pi or Pi
∞	infinity
$\sin^2(x) = (\sin(x))^2$	sin(x)^2 or (sin(x))^2

Notes

Maple TA likes to make the following substitutions when displaying equations

Simple Form	Maple TA Will Show
sec(x)^2	1+tan(x)^2
csc(x)^2	1+cot(x)^2
sec(x)*tan(x)	sin(x)/cos(x)^2
csc(x)*cot(x)	cos(x)/sin(x)^2

Introductory Mathematical Economics Sample Assignments

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Assignment 1: Algebra and Calculus Review

Question 1: *Score 1/1*

The expression $4\,X\,Y\left(\frac{7}{X}+4\,Y\right)$ simplifies to

Correct

Your Answer: $28\,Y+16\,X\,Y^2$

Comment: In general, $A(B+C)=AB+AC$.

Question 2: *Score 1/1*

Your response

Differentiate the following function with respect to X:

F(X)= $e^{(-4X^2-3)}$

$\frac{\partial F}{\partial X}$ = **-8*X*exp(-4*X^2-3)** (100%)

Correct

Comment:

In general, $\frac{d}{dx}e^{f(x)}=e^{f(x)}\cdot\frac{d(f(x))}{dx}$.

The derivative is:

$\frac{\partial F}{\partial X}$ = $-8\,X\,e^{(-4X^2-3)}$

Question 3: *Score 1/1*

The expression $\frac{72\,X\,Y+16\,X^2}{8\,Y}$ simplifies to

Correct

Your Answer: $9\,X+2\,\frac{X^2}{Y}$

Comment: In general, $\frac{A+B}{C}=\frac{A}{C}+\frac{B}{C}$.

Question 4: *Score 1/1*

All the items matched correctly.

Correct

Match	Your Choice	✔/✘
$(ln(a) + ln(b) + ln(c))\, d$	$ln(a^d\, b^d\, c^d)$	✔
$ln((a + b + c)^d)$	$(ln(a + b + c))*(d)$	✔
$ln((a + b + c)\, d)$	$ln(a + b + c) + ln(d)$	✔

Comment:

In general, $\ln(AB) = \ln(A) + \ln(B)$ and $\ln(A^B) = B\, ln(A)$.

Question 5: *Score 1/1*

Your response

Differentiate the following function with respect to X:

F(X)= $3 - 5\,X - 8\,X^2 - X^3$

$\frac{\partial F}{\partial X}$ = **-5-16*X-3*X^2** (100%)

Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = -5 - 16\,X - 3\,X^2$$

Question 6: *Score 1/1*

The expression $7\,e^{(4\,X\,Y + 9\,X^2)}$ is equal to

Correct

Your Answer: $7\,e^{(4\,X\,Y)}\,e^{(9\,X^2)}$

Comment: In general, $e^A e^B = e^{A+B}$.

Question 7: *Score 1/1*

All the items matched correctly.

Correct

Match	Your Choice	✔/✘
$ln\left(e^{(X+2)}\, e^{(X-2)}\right)$	$2\,X$	✔
$e^{((X+2)\, ln(X-2))}$	$(X-2)^{(X+2)}$	✔
$(X+2)\,(X-2)$	X^2-4	✔

Comment:

In general, $e^A e^B = e^{A+B}$ and $e^{\ln(A)} = A$.

Question 8: *Score 1/1*

Your response

Differentiate the following function with respect to X:

$F(X) = ln\left(7\,X^2 - 8\,X + 5\right)$

$\frac{\partial F}{\partial X}$ = **(14*X-8)/(7*X^2-8*X+5)** (100%)

Correct

Comment:

In general, $\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)}\frac{d(f(x))}{dx}$.

The derivative is:

$$\frac{\partial F}{\partial X} = \frac{14\,X - 8}{7\,X^2 - 8\,X + 5}$$

Question 9: *Score 1/1*

Your response

Differentiate the following function with respect to X:

F(X)= $e^{(6\,X^2-9)}$

$\frac{\partial F}{\partial X}$ = **12*X*exp(6*X^2-9)** (100%)

Correct

Comment:

In general, $\frac{d}{dx}e^{f(x)} = e^{f(x)}\cdot\frac{d(f(x))}{dx}$.

The derivative is:

$$\frac{\partial F}{\partial X} = 12\,X\,e^{(6\,X^2-9)}$$

Question 10: *Score 1/1*

The expression $8X(4+5Y)$ simplifies to

Correct

Your Answer: $32X+40XY$

Comment: In general, $A(B+C)=AB+AC$.

Assignment 2: Differentiation Rules

Question 1: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X : $F(X) = \left(-4 - 7X - 2X^2 - 7X^3\right)^3$ $\frac{\partial F}{\partial X}$ = **3*(-4-7*X-2*X^2-7*X^3)^2*(-7-4*X-21*X^2)** (100%)	Differentiate the following function with respect to X : $F(X) = \left(-4 - 7X - 2X^2 - 7X^3\right)^3$ $\frac{\partial F}{\partial X}$ = **3*(-4-7*X-2*X^2-7*X^3)^2*(-7-4*X-21*X^2)**	Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = 3\left(-4 - 7X - 2X^2 - 7X^3\right)^2\left(-7 - 4X - 21X^2\right)$$

Question 2: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X : $F(X) = \frac{-7X + 5}{9 + X^2}$ $\frac{\partial F}{\partial X}$ = **-7/(9+X^2)-2*(-7*X+5)/(9+X^2)^2*X** (100%)	Differentiate the following function with respect to X : $F(X) = \frac{-7X + 5}{9 + X^2}$ $\frac{\partial F}{\partial X}$ = **-7/(9+X^2)-2*(-7*X+5)/(9+X^2)^2*X**	Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = -\frac{7}{9 + X^2} - 2\frac{(-7X + 5)X}{\left(9 + X^2\right)^2}$$

Question 3: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X : $F(X) = \frac{e^{(-2-X^2)}}{-2X^2-3X-6}$ $\frac{\partial F}{\partial X}$ = **-2*X*exp(-2-X^2)/(-2*X^2-3*X-6)-exp(-2-X^2)/(-2*X^2-3*X-6)^2*(-4*X-3)** (100%)	Differentiate the following function with respect to X : $F(X) = \frac{e^{(-2-X^2)}}{-2X^2-3X-6}$ $\frac{\partial F}{\partial X}$ = **-2*X*exp(-2-X^2)/(-2*X^2-3*X-6)-exp(-2-X^2)/(-2*X^2-3*X-6)^2*(-4*X-3)**	Correct

Comment:

In general, $\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot \frac{d(f(x))}{dx}$.

The derivative is:

$$\frac{\partial F}{\partial X} = -2\frac{X e^{(-2-X^2)}}{-2X^2-3X-6} - \frac{e^{(-2-X^2)}(-4X-3)}{(-2X^2-3X-6)^2}$$

Question 4: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X : $F(X) = -6X^2-8-ln(6+8X^2)$ $\frac{\partial F}{\partial X}$ = **-12*X-16*X/(6+8*X^2)** (100%)	Differentiate the following function with respect to X : $F(X) = -6X^2-8-ln(6+8X^2)$ $\frac{\partial F}{\partial X}$ = **-12*X-16*X/(6+8*X^2)**	Correct

Comment:

In general, $\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)} \frac{d(f(x))}{dx}$.

The derivative is:

$$\frac{\partial F}{\partial X} = -12X - 16\frac{X}{6+8X^2}$$

Question 5: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X: $F(X) = (4X + 3)\ ln(-1 + 5X)$ $\frac{\partial F}{\partial X}$ = **4*ln(-1+5*X)+5*(4*X+3)/(-1+5*X)** (100%)	Differentiate the following function with respect to X: $F(X) = (4X + 3)\ ln(-1 + 5X)$ $\frac{\partial F}{\partial X}$ = **4*ln(-1+5*X)+5*(4*X+3)/(-1+5*X)**	Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = 4\ ln(-1 + 5X) + 5\ \frac{4X + 3}{-1 + 5X}$$

Question 6: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X: $F(X) = \frac{8 + X}{-4X^2 + 1}$ $\frac{\partial F}{\partial X}$ = **1/(-4*X^2+1)+8*(8+X)/(-4*X^2+1)^2*X** (100%)	Differentiate the following function with respect to X: $F(X) = \frac{8 + X}{-4X^2 + 1}$ $\frac{\partial F}{\partial X}$ = **1/(-4*X^2+1)+8*(8+X)/(-4*X^2+1)^2*X**	Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = \frac{1}{-4X^2 + 1} + 8\ \frac{(8 + X)\ X}{(-4X^2 + 1)^2}$$

Question 7: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X: $F(X) = e^{\left(-2X^2-3\right)} + ln\left(8 + 8X^2\right)$ $\frac{\partial F}{\partial X}$ = **-4*X*exp(-2*X^2-3)+16*X/(8+8*X^2)** (100%)	Differentiate the following function with respect to X: $F(X) = e^{\left(-2X^2-3\right)} + ln\left(8 + 8X^2\right)$ $\frac{\partial F}{\partial X}$ = **-4*X*exp(-2*X^2-3)+16*X/(8+8*X^2)**	Correct

Comment:

In general, $\frac{d\left(e^{f(x)}\right)}{dx} = e^{f(x)} \cdot \frac{df(x)}{dx}$, and $\frac{d}{dx}\ln(f(x)) = \frac{1}{f(x)} \cdot \frac{df(x)}{dx}$.

The derivative is:

$$\frac{\partial F}{\partial X} = -4\,X\,e^{\left(-2X^2-3\right)} + 16\,\frac{X}{8 + 8X^2}$$

Question 8: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X $F(X) = \left(7X^2 + 5\right) e^{\left(-1-5X^2\right)}$. **14*X*exp(-1-5*X^2)-10*(7*X^2+5)*X*exp(-1-5*X^2)** (100%)	Differentiate the following function with respect to X $F(X) = \left(7X^2 + 5\right) e^{\left(-1-5X^2\right)}$. **14*X*exp(-1-5*X^2)-10*(7*X^2+5)*X*exp(-1-5*X^2)**	Correct

Comment:

In general, $\frac{d\left(e^{f(x)}\right)}{dx} = e^{f(x)} \cdot \frac{df(x)}{dx}$.

The derivative is:

$$\frac{\partial F}{\partial X} = 14\,X\,e^{\left(-1-5X^2\right)} - 10\left(7X^2 + 5\right) X\,e^{\left(-1-5X^2\right)}$$

Question 9: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X: $F(X) = 2X^4 + 4X^3 + 3X^2 - 5X + 7$ $\frac{\partial F}{\partial X}$ = **8*X^3+12*X^2+6*X-5** (100%)	Differentiate the following function with respect to X: $F(X) = 2X^4 + 4X^3 + 3X^2 - 5X + 7$ $\frac{\partial F}{\partial X}$ = **8*X^3+12*X^2+6*X-5**	Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = 8X^3 + 12X^2 + 6X - 5$$

Question 10: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X: $F(X) = \frac{ln(2 - 2X^2)}{3X^2 + 7}$ $\frac{\partial F}{\partial X}$ = **-4*X/(2-2*X^2)/(3*X^2+7)-6*ln(2-2*X^2)/(3*X^2+7)^2*X** (100%)	Differentiate the following function with respect to X: $F(X) = \frac{ln(2 - 2X^2)}{3X^2 + 7}$ $\frac{\partial F}{\partial X}$ = **-4*X/(2-2*X^2)/(3*X^2+7)-6*ln(2-2*X^2)/(3*X^2+7)^2*X**	Correct

Comment:

The derivative is:

$$\frac{\partial F}{\partial X} = -4\frac{X}{(2 - 2X^2)(3X^2 + 7)} - 6\frac{ln(2 - 2X^2)X}{(3X^2 + 7)^2}$$

Assignment 3: Optimization, Cost Functions

Question 1: *Score 1/1*

Your response	Correct response	
A competitive firm faces a price of 136 and a total cost function of TC= $7q^2 + 6q + 5$. What is this firm's marginal cost function? MC(q)= **14*q+6** (50%) What quantity should this firm produce? Leave your answer in fraction form (if necessary). q= **65/7** (50%)	A competitive firm faces a price of 136 and a total cost function of TC= $7q^2 + 6q + 5$. What is this firm's marginal cost function? MC(q)= **14*q+6** What quantity should this firm produce? Leave your answer in fraction form (if necessary). q= **65/7**	Correct

Comment:

The marginal cost is the derivative of $7q^2 + 6q + 5$, which is $14q + 6$.
Setting P=MC, and solving for q, gives q=65/7.

Question 2: *Score 1/1*

Your response	Correct response	
If Inverse Demand is P= $2 - 0.13Q$, what is the formula for the price-elasticity of demand? (Round to two decimals.) $\varepsilon(P)$ = **-7.69*P/(15.38-7.69*P)** (33%) If the price is 1.2, what is the elasticity? (Round to the nearest quarter.) $\varepsilon(1.2)$ = **1.5** (33%) Therefore when the price is 1.2, demand is **Elastic** (33%)	If Inverse Demand is P= $2 - 0.13Q$, what is the formula for the price-elasticity of demand? (Round to two decimals.) $\varepsilon(P)$ = **-7.69*P/(15.38-7.69*P)** If the price is 1.2, what is the elasticity? (Round to the nearest quarter.) $\varepsilon(1.2)$ = **1.5** Therefore when the price is 1.2, demand is **Elastic**	Correct

Comment:

Remember, $\varepsilon = \frac{P}{Q}\frac{dQ}{dP}$.

Question 3: *Score 1/1*

Your response	Correct response	
Given the following function: $F(X) = -87 + 89X^2 + 86X$ Given the above first order condition, what is the value of the critical point? $Xcrit =$ **-43/89** (100%)	Given the following function: $F(X) = -87 + 89X^2 + 86X$ Given the above first order condition, what is the value of the critical point? $Xcrit =$ **-43/89**	Correct

Comment:

Please provide a single numerical response per answer field. To account for a small amount of rounding, there is a range of values around the correct answer that will be awarded full marks.

Question 4: *Score 1/1*

Your response	Correct response	
Given the following function: $F(X) = 6 + 67X^2 - 80X$ Find the first order condition for optimizing this function: $\frac{dF}{dX} =$ **134*X-80** (20%) = **0** (20%) Given the above first order condition, what is the value of the critical point? $Xcrit =$ **40/67** (20%) What is the second derivative of $F(X)$ with respect to X? $\frac{d^2F}{dX^2} =$ **134** (20%) Given the above second derivative, is the critical point a maximum, minimum or is it indeterminate? $Xcrit$ is a **min** (20%)	Given the following function: $F(X) = 6 + 67X^2 - 80X$ Find the first order condition for optimizing this function: $\frac{dF}{dX} =$ **134*X-80** = **0** Given the above first order condition, what is the value of the critical point? $Xcrit =$ **40/67** What is the second derivative of $F(X)$ with respect to X? $\frac{d^2F}{dX^2} =$ **134** Given the above second derivative, is the critical point a maximum, minimum or is it indeterminate? $Xcrit$ is a **min**	Correct

Comment:

Please provide a single numerical response per answer field. To account for a small amount of rounding, there is a range of values around the correct answer that will be awarded full marks.

Question 5: *Score 1/1*

Your response	Correct response	
A perfectly competitive firm faces a price of P=13 and has a total cost function of C= $3q^2 + 7q$. What quantity should the firm produce? (Round answer to two decimal places if necessary. For example, 1.6666 becomes 1.67.) q= **1.00** (100%)	A perfectly competitive firm faces a price of P=13 and has a total cost function of C= $3q^2 + 7q$. What quantity should the firm produce? (Round answer to two decimal places if necessary. For example, 1.6666 becomes 1.67.) q= **1.00**	Correct

Comment:

Please provide a single numerical response per answer field. To account for a small amount of rounding, there is a range of values around the correct answer that will be awarded full marks.

If the price is low enough, the firm can't find a positive quantity that equates marginal cost and price. Therefore when this happens, the firm is best to produce zero quantity.

Question 6: *Score 1/1*

Your response	Correct response	
A perfectly competitive firm faces a price of P=16 and has a total cost function of C= $2q^2 + 3q + 17$. What quantity should the firm produce in the short run? Remember that this means the firm still needs to pay the fixed part of its cost function. (Round answer to two decimal places if necessary. For example, 1.6666 becomes 1.67.) q SR= **3.25** (50%) What quantity should the firm produce in the long run? (Round to two decimal places.) q LR= **3.25** (50%)	A perfectly competitive firm faces a price of P=16 and has a total cost function of C= $2q^2 + 3q + 17$. What quantity should the firm produce in the short run? Remember that this means the firm still needs to pay the fixed part of its cost function. (Round answer to two decimal places if necessary. For example, 1.6666 becomes 1.67.) q SR= **3.25** What quantity should the firm produce in the long run? (Round to two decimal places.) q LR= **3.25**	**Correct**

Comment:

Please provide a single numerical response per answer field. To account for a small amount of rounding, there is a range of values around the correct answer that will be awarded full marks.

Question 7: *Score 1/1*

Your response	Correct response	
If Demand is Q= $12 - 2P$, what is the formula for the price-elasticity of demand? $\varepsilon(P)$ = **2.0*P/(12-2*P)** (33%) If the price is 2, what is the elasticity? (Round to the nearest quarter.) $\varepsilon(2)$ = **0.5** (33%) Therefore when the price is 2, demand is **Inelastic** (33%)	If Demand is Q= $12 - 2P$, what is the formula for the price-elasticity of demand? $\varepsilon(P)$ = **2.0*P/(12-2*P)** If the price is 2, what is the elasticity? (Round to the nearest quarter.) $\varepsilon(2)$ = **0.5** Therefore when the price is 2, demand is **Inelastic**	**Correct**

Comment:

Remember, $\varepsilon = \frac{P}{Q}\frac{dQ}{dP}$.

Question 8: *Score 1/1*

Your response	Correct response	
Given the following function: $F(X) = -93 - 31\,X^2 - 19\,X$ What is the value of the critical point for this function? $Xcrit =$ **-19/62** (50%) Is this critical point a maximum, minimum or is it indeterminate? $Xcrit$ is a **max** (50%)	Given the following function: $F(X) = -93 - 31\,X^2 - 19\,X$ What is the value of the critical point for this function? $Xcrit =$ **-19/62** Is this critical point a maximum, minimum or is it indeterminate? $Xcrit$ is a **max**	**Correct**

Comment:

Please provide a single numerical response per answer field. To account for a small amount of rounding, there is a range of values around the correct answer that will be awarded full marks.

Question 9: *Score 1/1*

Your response	Correct response	
To find the extrema of the function $F(X)$ we differentiate and set the derivative to zero, $F'(X) = 0$. Then we check the second order condition which for a minimum is that **F''(X)>0** (50%) is a **sufficient** (50%) condition.	To find the extrema of the function $F(X)$ we differentiate and set the derivative to zero, $F'(X) = 0$. Then we check the second order condition which for a minimum is that **F''(X)>0** is a **sufficient** condition.	**Correct**

Comment:

Question 10: *Score 1/1*

Your response	Correct response	
A perfectly competitive firm faces a price of P=9 and has a total cost function of C= $2\,q^2 + 2\,q + 15$. What quantity should the firm produce in the short run? Remember that this means the firm still needs to pay the fixed part of its cost function. (Round answer to two decimal places if necessary. For example, 1.6666 becomes 1.67.) q SR= **1.75** (50%) What quantity should the firm produce in the long run? (If now the firm has the option of shutting down and paying no cost.) (Round to two decimal places.) q LR= **0.0** (50%)	A perfectly competitive firm faces a price of P=9 and has a total cost function of C= $2\,q^2 + 2\,q + 15$. What quantity should the firm produce in the short run? Remember that this means the firm still needs to pay the fixed part of its cost function. (Round answer to two decimal places if necessary. For example, 1.6666 becomes 1.67.) q SR= **1.75** What quantity should the firm produce in the long run? (If now the firm has the option of shutting down and paying no cost.) (Round to two decimal places.) q LR= **0.0**	**Correct**

Comment:

In the short run, it may still be profitable for the firm to produce some quantity even though it is making negative profit. This is because its only alternative is to make -15.

However, if the price is so low that the firm cannot even equate MC=P, it is optimal for the firm to produce zero in both the short run and the long run.

Assignment 4: Systems of Equations and Matrices

Question 1: *Score 1/1*

Your response

Rearrange the following system of equations to be in the form $a\,x + b\,y + c\,z = constant$, where $a \geq 0$.

$3\,y + 8\,x + 2\,z = 9$

$-1 + 4\,z = -10\,x - 2\,y$

$2\,x + 6\,y - 7\,z = 10$

(If the coefficient is zero, enter 0. If the coefficient is one, enter 1. Enter all other numbers normally.)

8 (8%) x+ **3** (8%) y+ **2** (8%) z= **9** (8%)

10 (8%) x+ **2** (8%) y+ **4** (8%) z= **1** (8%)

2 (8%) x+ **6** (8%) y+ **-7** (8%) z= **10** (8%)

Correct response

Rearrange the following system of equations to be in the form $a\,x + b\,y + c\,z = constant$, where $a \geq 0$.

$3\,y + 8\,x + 2\,z = 9$

$-1 + 4\,z = -10\,x - 2\,y$

$2\,x + 6\,y - 7\,z = 10$

(If the coefficient is zero, enter 0. If the coefficient is one, enter 1. Enter all other numbers normally.)

8 x+ **3** y+ **2** z= **9**

10 x+ **2** y+ **4** z= **1**

2 x+ **6** y+ **-7** z= **10**

Correct

Comment:

The rearranged system looks like this:

$8\,x + 3\,y + 2\,z = 9$

$10\,x + 2\,y + 4\,z = 1$

$2\,x + 6\,y - 7\,z = 10$

Question 2: *Score 1/1*

Give the coefficient matrix of the following system. Use the order X, Y, Z.

$5\,Z - 5\,X + 7\,Y = -4$

$-2\,Z - 6\,Y = -1$

$-7\,Y + 3\,X + Z = 1$

Correct

Your Answer:

$$\begin{bmatrix} -5 & 7 & 5 \\ 0 & -6 & -2 \\ 3 & -7 & 1 \end{bmatrix}$$

Correct Answer:

$$\begin{pmatrix} -5 & 7 & 5 \\ 0 & -6 & -2 \\ 3 & -7 & 1 \end{pmatrix}$$

Comment: Make sure you enter the coefficients in the order X, Y, Z in each row of the matrix. Don't forget the sign.

Question 3: *Score 1/1*

The following system of equations

$3x + 4y = 6$

$8y + 6x = 4$

Correct

Your Answer: has no solution

Correct Answer: has no solution

Comment:

In general, if you encounter a contradiction (eg. $0 = 1$) then there is no solution.

Question 4: *Score 1/1*

Your response

Consider the following system of equations:

$-3 + 3Y = 0$

$-3X + Y + 6Z + 5 = 0$

$-2X + 3Y + 4Z + 1 = 0$

Choose the answer that correctly describes the system

The system **has an infinite number of solutions.** (25%) .

Give the solutions for the system, if they exist.

If there is no solution type "no solution" (without quotes) in each case.

If there are many solutions, for a two parameter solution type Y in the $Y=$ box and Z in the $Z=$ box, and then give X in terms of Y and Z;

For a one parameter solution type Z in the $Z=$ box and then give X and Y in terms of Z.

$X=$ **2+2*Z** (25%) , $Y=$ **1** (25%) , and $Z=$ **z** (25%)

Correct response

Consider the following system of equations:

$-3 + 3Y = 0$

$-3X + Y + 6Z + 5 = 0$

$-2X + 3Y + 4Z + 1 = 0$

Choose the answer that correctly describes the system

The system **has an infinite number of solutions.** .

Give the solutions for the system, if they exist.

If there is no solution type "no solution" (without quotes) in each case.

If there are many solutions, for a two parameter solution type Y in the $Y=$ box and Z in the $Z=$ box, and then give X in terms of Y and Z;

For a one parameter solution type Z in the $Z=$ box and then give X and Y in terms of Z.

$X=$ **2+2*Z** , $Y=$ **1** , and $Z=$ **z**

Correct

Comment:

This system has multiple solutions with one parameter. This can be seen using a variety of methods. One method is to form the 3x4 matrix of coefficients.

$$\begin{pmatrix} 0 & 3 & 0 & 3 \\ -3 & 1 & 6 & -5 \\ -2 & 3 & 4 & -1 \end{pmatrix}$$

Then apply elementary row operation until the row echelon form is achieved.

$$\begin{pmatrix} 1 & 0 & -2 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Now the answers can be simply read from the matrix.

Question 5: *Score 1/1*

The following system of equations

$5x + 4y = 8$

$8y + 10x = 2$

Correct

Your Answer: has no solution

Correct Answer: has no solution

Comment: This system of equations has no solutions because looking at the left hand sides of the two equations the second is two times the first; however, looking at the right hand sides the second equation has 2 which is certainly not two times 8, the right hand side of the first equation.

Question 6: *Score 1/1*

Your response	Correct response
Consider the following system of equations: $-2X + 11 - 3Y + 3Z = 0$ $-2X + 31 - 2Y - Z = 0$ $Y - 8 = 0$ Choose the answer that correctly describes the system The system **has a unique solution** (25%) . Give the solutions for the system, if they exist. If there is no solution type "no solution" (without quotes) in each case. If there are many solutions, for a two parameter solution type Y in the $Y=$ box and Z in the $Z=$ box, and then give X in terms of Y and Z. For one parameter solutions type Z in the $Z=$ box and then give X and Y in terms of Z. $X=$ **4** (25%) , $Y=$ **8** (25%) , and $Z=$ **7** (25%)	Consider the following system of equations: $-2X + 11 - 3Y + 3Z = 0$ $-2X + 31 - 2Y - Z = 0$ $Y - 8 = 0$ Choose the answer that correctly describes the system The system **has a unique solution** . Give the solutions for the system, if they exist. If there is no solution type "no solution" (without quotes) in each case. If there are many solutions, for a two parameter solution type Y in the $Y=$ box and Z in the $Z=$ box, and then give X in terms of Y and Z. For one parameter solutions type Z in the $Z=$ box and then give X and Y in terms of Z. $X=$ **4** , $Y=$ **8** , and $Z=$ **7**

Correct

Comment:

There are various ways to solve this question.

One way is to take the matrix of coefficients $\begin{pmatrix} -2 & -3 & 3 \\ -2 & -2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ and combine it with the column of constants $\begin{pmatrix} -11 \\ -31 \\ 8 \end{pmatrix}$, to form a 3x4 matrix. Then use elementary row operations to reduce the matrix to its Row Echelon Form $\begin{pmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7 \end{pmatrix}$. Now we can just read the solutions from the matrix: $X=$ 4, $Y=$ 8, and $Z=$ 7.

Question 7: *Score 1/1*

A system of equations can be written in the matrix form $Ax = b$. Give the coefficient matrix of the following system. Use the order $x_1 \ x_2 \ x_3$.

$$2x_2 + x_1 = 1 + 3\,x_3$$

$$2\,x_3 + 7\,x_1 = x_2 + 4$$

Correct

Your Answer: $\begin{bmatrix} 1 & 2 & -3 \\ 7 & -1 & 2 \end{bmatrix}$

Correct Answer: $\begin{pmatrix} 1 & 2 & -3 \\ 7 & -1 & 2 \end{pmatrix}$

Comment:

Question 8: *Score 1/1*

Your response	Correct response
Solve for $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$:	Solve for $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$:
$\begin{pmatrix} 0 & 0 & -2 \\ 1 & -1 & -3 \\ -3 & 2 & -3 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 4 \\ -5 \\ 33 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & -2 \\ 1 & -1 & -3 \\ -3 & 2 & -3 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 4 \\ -5 \\ 33 \end{pmatrix}$
$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \\ -2 \end{bmatrix}$ (100%)	$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 6 \\ -2 \end{bmatrix}$

Correct

Comment:

$$x = \begin{pmatrix} -5 \\ 6 \\ -2 \end{pmatrix}$$

Question 9: *Score 1/1*

Your response	Correct response	
Solve for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$: $\begin{pmatrix} 4 & -10 \\ 4 & 7 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} 100 \\ -36 \end{pmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$ (100%)	Solve for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$: $\begin{pmatrix} 4 & -10 \\ 4 & 7 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{pmatrix} 100 \\ -36 \end{pmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix}$	Correct

Comment:

x= $\begin{pmatrix} 5 \\ -8 \end{pmatrix}$

Question 10: *Score 1/1*

Your response	Correct response	
Translate the following system of equations into a Matrix A, where $A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ equals the system of equations. $-6x + 8y - 6z = 0$ $9x + 5y - 8z = 0$ $8x + 4y + 10z = 0$ (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix.) $A = \begin{bmatrix} -6 & 8 & -6 \\ 9 & 5 & -8 \\ 8 & 4 & 10 \end{bmatrix}$ (100%)	Translate the following system of equations into a Matrix A, where $A \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ equals the system of equations. $-6x + 8y - 6z = 0$ $9x + 5y - 8z = 0$ $8x + 4y + 10z = 0$ (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix.) $A = \begin{bmatrix} -6 & 8 & -6 \\ 9 & 5 & -8 \\ 8 & 4 & 10 \end{bmatrix}$	Correct

Comment:

Make sure you enter the coefficients in the order x, y, z in each row of the matrix. Don't forget the sign.

Assignment 5: Matrix Properties

Question 1: *Score 1/1*

Consider the following matrix.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -6 & 6 \\ 0 & -7 & 7 \end{pmatrix}$$

Choose all the terms that apply to this matrix.

Choice	Selected	✔/✘	Points
diagonal matrix	No		
zero matrix	No		
idempotent	Yes	✔	+1
symmetric	No		
negative definite	No		
identity	No		
positive definite	No		
invertible	No		
indefinite	No		
triangular	No		
full rank	No		

Correct

Number of available correct choices: 1

Partial Grading Explained

Comment:

Question 2: *Score 1/1*

Your response	Correct response
Suppose A and B are two non-singular matrices. $(A \cdot B)^T$ is equal to $B^T \cdot A^T$ (100%)	Suppose A and B are two non-singular matrices. $(A \cdot B)^T$ is equal to $B^T \cdot A^T$

Correct

Comment:

Question 3: *Score 1/1*

Your response	Correct response	
Calculate the following: (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $3\begin{pmatrix} 2 & -2 \\ -2 & 1 \\ -4 & 2 \\ -4 & -5 \end{pmatrix} - 5\begin{pmatrix} -1 & -4 \\ 1 & -2 \\ -2 & -5 \\ 5 & 2 \end{pmatrix} = \begin{bmatrix} 11 & 14 \\ -11 & 13 \\ -2 & 31 \\ -37 & -25 \end{bmatrix}$ (100%)	Calculate the following: (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $3\begin{pmatrix} 2 & -2 \\ -2 & 1 \\ -4 & 2 \\ -4 & -5 \end{pmatrix} - 5\begin{pmatrix} -1 & -4 \\ 1 & -2 \\ -2 & -5 \\ 5 & 2 \end{pmatrix} = \begin{bmatrix} 11 & 14 \\ -11 & 13 \\ -2 & 31 \\ -37 & -25 \end{bmatrix}$	Correct

Comment:

$$3\begin{pmatrix} 2 & -2 \\ -2 & 1 \\ -4 & 2 \\ -4 & -5 \end{pmatrix} - 5\begin{pmatrix} -1 & -4 \\ 1 & -2 \\ -2 & -5 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 11 & 14 \\ -11 & 13 \\ -2 & 31 \\ -37 & -25 \end{pmatrix}$$

Question 4: *Score 1/1*

Your response	Correct response	
Compute A times B. $A=\begin{pmatrix} -2 & 3 \\ -1 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 0 & 2 & -2 \\ 3 & 2 & -1 \end{pmatrix}$ (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $AB=\begin{bmatrix} 9 & 2 & 1 \\ -3 & -4 & 3 \end{bmatrix}$ (100%)	Compute A times B. $A=\begin{pmatrix} -2 & 3 \\ -1 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 0 & 2 & -2 \\ 3 & 2 & -1 \end{pmatrix}$ (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $AB=\begin{bmatrix} 9 & 2 & 1 \\ -3 & -4 & 3 \end{bmatrix}$	Correct

Comment:

$$AB=\begin{pmatrix} 9 & 2 & 1 \\ -3 & -4 & 3 \end{pmatrix}$$

Question 5: *Score 1/1*

Your response	Correct response
Calculate the following: (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $\begin{pmatrix} 3 & 3 & 4 & -5 \\ 4 & -3 & -2 & -2 \\ 4 & 0 & 1 & 1 \\ -3 & 4 & 3 & -2 \end{pmatrix} + \begin{pmatrix} -1 & 2 & -3 & -1 \\ 3 & 4 & 0 & 2 \\ -5 & 2 & 3 & 2 \\ 2 & -4 & -3 & 0 \end{pmatrix} =$ $\begin{bmatrix} 2 & 5 & 1 & -6 \\ 7 & 1 & -2 & 0 \\ -1 & 2 & 4 & 3 \\ -1 & 0 & 0 & -2 \end{bmatrix}$ (100%)	Calculate the following: (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $\begin{pmatrix} 3 & 3 & 4 & -5 \\ 4 & -3 & -2 & -2 \\ 4 & 0 & 1 & 1 \\ -3 & 4 & 3 & -2 \end{pmatrix} + \begin{pmatrix} -1 & 2 & -3 & -1 \\ 3 & 4 & 0 & 2 \\ -5 & 2 & 3 & 2 \\ 2 & -4 & -3 & 0 \end{pmatrix} =$ $\begin{bmatrix} 2 & 5 & 1 & -6 \\ 7 & 1 & -2 & 0 \\ -1 & 2 & 4 & 3 \\ -1 & 0 & 0 & -2 \end{bmatrix}$

Correct

Comment:

$$\begin{pmatrix} 3 & 3 & 4 & -5 \\ 4 & -3 & -2 & -2 \\ 4 & 0 & 1 & 1 \\ -3 & 4 & 3 & -2 \end{pmatrix} + \begin{pmatrix} -1 & 2 & -3 & -1 \\ 3 & 4 & 0 & 2 \\ -5 & 2 & 3 & 2 \\ 2 & -4 & -3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 5 & 1 & -6 \\ 7 & 1 & -2 & 0 \\ -1 & 2 & 4 & 3 \\ -1 & 0 & 0 & -2 \end{pmatrix}$$

Question 6: *Score 1/1*

Your response	Correct response	
Find AB. $A=\begin{pmatrix} 3 & -1 & 0 \\ -3 & 2 & 3 \\ 0 & 3 & -3 \end{pmatrix}$ $B=\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$ If the matrices do not conform, type DNC. (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $\begin{bmatrix} 0 \\ -9 \\ 9 \end{bmatrix}$ (100%) Note: Please enter the values exactly - ie. as fractions or whole numbers, not as decimal numbers.	Find AB. $A=\begin{pmatrix} 3 & -1 & 0 \\ -3 & 2 & 3 \\ 0 & 3 & -3 \end{pmatrix}$ $B=\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$ If the matrices do not conform, type DNC. (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $\begin{bmatrix} 0 \\ -9 \\ 9 \end{bmatrix}$ Note: Please enter the values exactly - ie. as fractions or whole numbers, not as decimal numbers.	Correct

Comment:

$AB=\begin{pmatrix} 0 \\ -9 \\ 9 \end{pmatrix}$

Question 7: *Score 1/1*

Your response	Correct response	
Suppose A and B are two non-singular matrices, and their sum is also non-singular. $(A+B)^{-1}$ is equal to $(A+B)^{-1}$. *It cannot be simplified.* (100%)	Suppose A and B are two non-singular matrices, and their sum is also non-singular. $(A+B)^{-1}$ is equal to $(A+B)^{-1}$. *It cannot be simplified.*	Correct

Comment:

Question 8: *Score 1/1*

Consider the following matrix.

$$\begin{pmatrix} -2 & -69 & -42 \\ -72 & -60 & -72 \\ 86 & 123 & 126 \end{pmatrix}$$

Choose all the terms that apply to this matrix.

Choice	Selected	✔/✘	Points
indefinite	Yes	✔	+1
idempotent	No		
zero matrix	No		
identity	No		
symmetric	No		
positive definite	No		
negative definite	No		
full rank	Yes	✔	+1
triangular	No		
diagonal matrix	No		
invertible	Yes	✔	+1

✔ **Correct**

Number of available correct choices: 3

Partial Grading Explained

Comment:

Question 9: *Score 1/1*

Your response	Correct response	
$\begin{pmatrix} 2 & -3 & 3 \\ -3 & -1 & 1 \\ 2 & 1 & -2 \end{pmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 0 \\ -11 \\ 5 \end{pmatrix}$ Use Gauss Jordan to solve the above system of equations. You only need to transform your matrix to the point where it looks like this: $\begin{bmatrix} 1 & \# & \# & \# \\ 0 & 1 & \# & \# \\ \# & \# & \# & \# \end{bmatrix}$ (# means the values that result from row operations) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ (100%) Note: Your answer should be a 3x4 matrix.	$\begin{pmatrix} 2 & -3 & 3 \\ -3 & -1 & 1 \\ 2 & 1 & -2 \end{pmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 0 \\ -11 \\ 5 \end{pmatrix}$ Use Gauss Jordan to solve the above system of equations. You only need to transform your matrix to the point where it looks like this: $\begin{bmatrix} 1 & \# & \# & \# \\ 0 & 1 & \# & \# \\ \# & \# & \# & \# \end{bmatrix}$ (# means the values that result from row operations) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$ Note: Your answer should be a 3x4 matrix.	Correct

Comment:

Note: There is no *unique* solution to this problem.

For this question we need to do row operations until we have zeros and ones in the required locations. This means that you can interchange rows or add any multiple of a row to another row or multiply a row by a constant.

If you were to solve the system completely, the solution would be $\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{pmatrix}$.

Question 10: *Score 1/1*

Your response	Correct response	
Find the transpose of the following matrix: $A = \begin{pmatrix} 6 & 1 & 6 \\ 5 & 4 & -7 \\ 6 & -7 & -2 \end{pmatrix}$ (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $A^T = \begin{bmatrix} 6 & 5 & 6 \\ 1 & 4 & -7 \\ 6 & -7 & -2 \end{bmatrix}$ (100%)	Find the transpose of the following matrix: $A = \begin{pmatrix} 6 & 1 & 6 \\ 5 & 4 & -7 \\ 6 & -7 & -2 \end{pmatrix}$ (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $A^T = \begin{bmatrix} 6 & 5 & 6 \\ 1 & 4 & -7 \\ 6 & -7 & -2 \end{bmatrix}$	Correct

Comment:

Assignment 6: The Matrix Determinant and Inverse

Question 1: *Score 1/1*

Your response

What is the determinant of the following matrix?

$$A=\begin{pmatrix} 2 & 0 & -3 \\ -2 & 0 & -2 \\ -2 & 2 & 3 \end{pmatrix}$$

det(A)= **20** (50%)

What is the inverse of A?

(To enter your answer, right-click on the equation editor, select and then to bring up an empty matrix. Next, replace each letter with your numeric answer, tabbing between cells.)

$$\begin{bmatrix} \frac{1}{5} & -\frac{3}{10} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{5} & -\frac{1}{5} & 0 \end{bmatrix}$$ (50%)

Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

Correct response

What is the determinant of the following matrix?

$$A=\begin{pmatrix} 2 & 0 & -3 \\ -2 & 0 & -2 \\ -2 & 2 & 3 \end{pmatrix}$$

det(A)= **20**

What is the inverse of A?

(To enter your answer, right-click on the equation editor, select and then to bring up an empty matrix. Next, replace each letter with your numeric answer, tabbing between cells.)

$$\begin{bmatrix} \frac{1}{5} & -\frac{3}{10} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{5} & -\frac{1}{5} & 0 \end{bmatrix}$$

Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

Correct

Comment:

The determinant is 20.

The inverse is $\begin{pmatrix} \frac{1}{5} & \frac{-3}{10} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{-1}{5} & \frac{-1}{5} & 0 \end{pmatrix}$.

Question 2: *Score 1/1*

Your response	Correct response

Your response

What is the determinant of the following matrix?

$$\begin{pmatrix} -7 & 0 \\ 9 & 7 \end{pmatrix}$$

det(A)= **-49** (50%)

What is the inverse of A?

(To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.)

$$\begin{bmatrix} -\frac{1}{7} & 0 \\ \frac{9}{49} & \frac{1}{7} \end{bmatrix}$$ (50%)

Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

Correct response

What is the determinant of the following matrix?

$$\begin{pmatrix} -7 & 0 \\ 9 & 7 \end{pmatrix}$$

det(A)= **-49**

What is the inverse of A?

(To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.)

$$\begin{bmatrix} -\frac{1}{7} & 0 \\ \frac{9}{49} & \frac{1}{7} \end{bmatrix}$$

Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

Correct

Comment:

The determinant is -49.

The inverse is $\begin{pmatrix} \frac{-1}{7} & 0 \\ \frac{9}{49} & \frac{1}{7} \end{pmatrix}$.

Question 3: *Score 1/1*

Your response	Correct response
Julie is trying to make some bread and cake and uses an input requirement matrix to help her figure out how much she can make. An input requirement matrix represents how many inputs are needed to make one unit of different outputs. For example, the following matrix: $\begin{pmatrix} 4 & 3 \\ 4 & 2 \end{pmatrix}$ says that it takes Julie 4 cups of flour and 4 cups of sugar to make one loaf of bread. Similarly, it takes her 3 cups of flour and 2 cups of sugar to make one cake. (Julie is not a very good baker and therefore doesn't know the proper ratios.) If Julie has **42 units of flour** and **36 units of sugar**, how much bread and cake can she make? (Do not enter units.) Cake= **6** (50%) Bread= **6** (50%)	Julie is trying to make some bread and cake and uses an input requirement matrix to help her figure out how much she can make. An input requirement matrix represents how many inputs are needed to make one unit of different outputs. For example, the following matrix: $\begin{pmatrix} 4 & 3 \\ 4 & 2 \end{pmatrix}$ says that it takes Julie 4 cups of flour and 4 cups of sugar to make one loaf of bread. Similarly, it takes her 3 cups of flour and 2 cups of sugar to make one cake. (Julie is not a very good baker and therefore doesn't know the proper ratios.) If Julie has **42 units of flour** and **36 units of sugar**, how much bread and cake can she make? (Do not enter units.) Cake= **6** Bread= **6**

Correct

Comment:

The inverse of the input requirements matrix is:

$$\begin{pmatrix} \frac{-1}{2} & \frac{3}{4} \\ 1 & -1 \end{pmatrix}$$

and

$$\begin{pmatrix} \frac{-1}{2} & \frac{3}{4} \\ 1 & -1 \end{pmatrix} * \begin{pmatrix} 42 \\ 36 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

Therefore the bakery can make 6 loaves of bread and 6 cakes.

Question 4: *Score 1/1*

Your response	Correct response
If $\lvert A \rvert$ =-31 and $\lvert B \rvert$ =-6, What is the determinant of AB? $\lvert AB \rvert$ = **186** (100%)	If $\lvert A \rvert$ =-31 and $\lvert B \rvert$ =-6, What is the determinant of AB? $\lvert AB \rvert$ = **186**

Correct

Comment:

The determinant of A is -31.

The determinant of B is -6.

Therefore the determinant of AB is 186.

Question 5: *Score 1/1*

Your response	Correct response	
If the determinant of A= $\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ is 8, what is the determinant of B= $\begin{pmatrix} g & d & a \\ h & e & b \\ i & f & c \end{pmatrix}$?	If the determinant of A= $\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ is 8, what is the determinant of B= $\begin{pmatrix} g & d & a \\ h & e & b \\ i & f & c \end{pmatrix}$?	Correct
det(B)= **-8** (100%)	det(B)= **-8**	

Comment:

B is A, but with two columns switched. Therefore, the determinant of B is -det(A).

Question 6: *Score 1/1*

Your response	Correct response	
What is the determinant of the following matrix?	What is the determinant of the following matrix?	
A= $\begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 2 \\ 2 & -1 & 1 \end{pmatrix}$	A= $\begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 2 \\ 2 & -1 & 1 \end{pmatrix}$	Correct
det(A)= **-6** (50%)	det(A)= **-6**	
Is A invertible?	Is A invertible?	
Yes (50%)	**Yes**	

Comment:

The determinant of $\begin{pmatrix} -1 & -1 & 0 \\ 0 & 0 & 2 \\ 2 & -1 & 1 \end{pmatrix}$ is -6.

Because the determinant is not equal to zero, A is invertible.

Question 7: *Score 1/1*

Your response	Correct response	
If the determinant of A= $\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ is 5, what is the determinant of B= $\begin{pmatrix} 4a & 4d & 4g \\ 4b & 4e & 4h \\ 4c & 4f & 4i \end{pmatrix}$? det(B)= **320** (100%)	If the determinant of A= $\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix}$ is 5, what is the determinant of B= $\begin{pmatrix} 4a & 4d & 4g \\ 4b & 4e & 4h \\ 4c & 4f & 4i \end{pmatrix}$? det(B)= **320**	Correct

Comment:

B is A, but with every element multiplied by 4. Therefore, the determinant of B is 4^3 det(A)=320. Note that if the matrix was $n \times n$, the determinant would be 4^n det(A).

Question 8: *Score 1/1*

Your response	Correct response	
What is the determinant of the following matrix? A= $\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ -3 & -1 & 0 & -3 & -1 \\ 3 & 1 & 1 & 3 & 2 \\ 2 & -1 & -3 & 2 & 0 \\ -1 & 1 & 2 & 3 & 1 \end{pmatrix}$ det(A)= **-14** (100%)	What is the determinant of the following matrix? A= $\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ -3 & -1 & 0 & -3 & -1 \\ 3 & 1 & 1 & 3 & 2 \\ 2 & -1 & -3 & 2 & 0 \\ -1 & 1 & 2 & 3 & 1 \end{pmatrix}$ det(A)= **-14**	Correct

Comment:

The determinant is -14.

Question 9: *Score 1/1*

Your response	Correct response

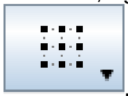
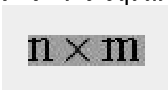
Your response

What is the determinant of the following matrix?

$$A=\begin{pmatrix} 1 & -3 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & -2 & -3 & 0 \\ 0 & -3 & 0 & -1 \end{pmatrix}$$

det(A)= **15** (50%)

What is the inverse of A?

(To enter your answer, right-click on the equation editor, select , then 

and then choose 4x4 on this screen: 

to bring up an empty matrix. Next, replace each letter with your numeric answer, tabbing between cells.)

$$\begin{bmatrix} -\frac{1}{5} & \frac{4}{3} & -\frac{1}{15} & \frac{17}{15} \\ -\frac{1}{5} & \frac{1}{3} & -\frac{1}{15} & \frac{2}{15} \\ 0 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{3}{5} & -1 & \frac{1}{5} & -\frac{7}{5} \end{bmatrix}$$ (50%)

Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

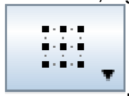
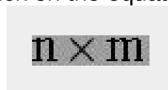
Correct response

What is the determinant of the following matrix?

$$A=\begin{pmatrix} 1 & -3 & 1 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & -2 & -3 & 0 \\ 0 & -3 & 0 & -1 \end{pmatrix}$$

det(A)= **15**

What is the inverse of A?

(To enter your answer, right-click on the equation editor, select , then 

and then choose 4x4 on this screen: 

to bring up an empty matrix. Next, replace each letter with your numeric answer, tabbing between cells.)

$$\begin{bmatrix} -\frac{1}{5} & \frac{4}{3} & -\frac{1}{15} & \frac{17}{15} \\ -\frac{1}{5} & \frac{1}{3} & -\frac{1}{15} & \frac{2}{15} \\ 0 & \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{3}{5} & -1 & \frac{1}{5} & -\frac{7}{5} \end{bmatrix}$$

Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

Correct

Comment:

The determinant is 15.

The inverse is $\begin{pmatrix} \frac{-1}{5} & \frac{4}{3} & \frac{-1}{15} & \frac{17}{15} \\ \frac{-1}{5} & \frac{1}{3} & \frac{-1}{15} & \frac{2}{15} \\ 0 & \frac{2}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{3}{5} & -1 & \frac{1}{5} & \frac{-7}{5} \end{pmatrix}$.

Question 10: *Score 1/1*

Your response	Correct response
What is the determinant of the following matrix? $\begin{pmatrix} 9 & -3 \\ 4 & 0 \end{pmatrix}$ det(A)= **12** (50%) What is the inverse of A? (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $\begin{bmatrix} 0 & \frac{1}{4} \\ -\frac{1}{3} & \frac{3}{4} \end{bmatrix}$ (50%) Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.	What is the determinant of the following matrix? $\begin{pmatrix} 9 & -3 \\ 4 & 0 \end{pmatrix}$ det(A)= **12** What is the inverse of A? (To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.) $\begin{bmatrix} 0 & \frac{1}{4} \\ -\frac{1}{3} & \frac{3}{4} \end{bmatrix}$ Note: Please enter the values *exactly* - ie. as fractions or whole numbers, not as decimal numbers.

Correct

Comment:

The determinant is 12.

The inverse is $\begin{pmatrix} 0 & \frac{1}{4} \\ \frac{-1}{3} & \frac{3}{4} \end{pmatrix}$.

Assignment 7: Partial Differentiation and Marginal Product

Question 1: *Score 1/1*

Your response	Correct response	
If the production function is $3\,L^{\frac{1}{8}}\,K^{\frac{1}{7}}$, what is the marginal rate of technical substitution for labour (L) with respect to capital (K)? $MRTS_{LK}$= **(8/7)*((L)/(K))** (100%)	If the production function is $3\,L^{\frac{1}{8}}\,K^{\frac{1}{7}}$, what is the marginal rate of technical substitution for labour (L) with respect to capital (K)? $MRTS_{LK}$= **(8/7)*((L)/(K))**	Correct

Comment:

The MRTS(L,K)= $\frac{MPK}{MPL} = \frac{8}{7}\,\frac{L}{K}$.

Question 2: *Score 1/1*

Your response	Correct response	
Differentiate $\frac{13}{18}\,X^2 + \frac{48}{29}\,X\,Y^3 + \frac{3}{2}\,Y^2$ with respect to Y. **144/29*X*Y^2+3*Y** (100%)	Differentiate $\frac{13}{18}\,X^2 + \frac{48}{29}\,X\,Y^3 + \frac{3}{2}\,Y^2$ with respect to Y. **144/29*X*Y^2+3*Y**	Correct

Comment:

Remember to treat X as a constant.

Question 3: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to Z: $F(X, Y, Z) =$ $X^5\,Z^5 + X^6\,Y^8\,Z^5 + 16\,Y^3\,Z^5 + Z^7\,X$ $\frac{\partial F}{\partial Z} =$ **5*X^5*Z^4+5*X^6*Y^8*Z^4+80*Y^3*Z^4+7*Z^6*X** (100%)	Differentiate the following function with respect to Z: $F(X, Y, Z) =$ $X^5\,Z^5 + X^6\,Y^8\,Z^5 + 16\,Y^3\,Z^5 + Z^7\,X$ $\frac{\partial F}{\partial Z} =$ **5*X^5*Z^4+5*X^6*Y^8*Z^4+80*Y^3*Z^4+7*Z^6*X**	Correct

Comment:

Remember to treat the other two variables as constants.
The derivative is:

$$\frac{\partial F}{\partial X} = 5\,X^5\,Z^4 + 5\,X^6\,Y^8\,Z^4 + 80\,Y^3\,Z^4 + 7\,Z^6\,X$$

Question 4: *Score 1/1*

Your response	Correct response	
Partially differentiate the following function: $F(X, Y, Z) = 6\,X^3\,Y + 7\,Y^4 - 8\,X\,Y\,Z + Z^2$ With respect to X: $\frac{\partial F}{\partial X}$ = **18*X^2*Y-8*Y*Z** (33%) With respect to Y: $\frac{\partial F}{\partial Y}$ = **6*X^3+28*Y^3-8*X*Z** (33%) With respect to Z: $\frac{\partial F}{\partial Z}$ = **-8*X*Y+2*Z** (33%)	Partially differentiate the following function: $F(X, Y, Z) = 6\,X^3\,Y + 7\,Y^4 - 8\,X\,Y\,Z + Z^2$ With respect to X: $\frac{\partial F}{\partial X}$ = **18*X^2*Y-8*Y*Z** With respect to Y: $\frac{\partial F}{\partial Y}$ = **6*X^3+28*Y^3-8*X*Z** With respect to Z: $\frac{\partial F}{\partial Z}$ = **-8*X*Y+2*Z**	Correct

Comment:

Remember to treat the other two variables in each case as constants.

Question 5: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to X: $F(X, Y) = 2\,X^9 + 11\,X^4\,Y^8 + 4\,Y^4$ $\frac{\partial F}{\partial X}$ = **18*X^8+44*X^3*Y^8** (100%)	Differentiate the following function with respect to X: $F(X, Y) = 2\,X^9 + 11\,X^4\,Y^8 + 4\,Y^4$ $\frac{\partial F}{\partial X}$ = **18*X^8+44*X^3*Y^8**	Correct

Comment:

Remember to treat Y as a constant.

The derivative is:

$$\frac{\partial F}{\partial X} = 18\,X^8 + 44\,X^3\,Y^8$$

Question 6: *Score 1/1*

Your response	Correct response	
Differentiate the following function with respect to Y: $F(X, Y, Z) =$ $X^4 Z^5 + 20 X^2 Y^7 Z^3 + 6 Y^6 Z^9 + Z^3 X$ $\frac{\partial F}{\partial Y}$ = **140*X^2*Y^6*Z^3+36*Y^5*Z^9** (100%)	Differentiate the following function with respect to Y: $F(X, Y, Z) =$ $X^4 Z^5 + 20 X^2 Y^7 Z^3 + 6 Y^6 Z^9 + Z^3 X$ $\frac{\partial F}{\partial Y}$ = **140*X^2*Y^6*Z^3+36*Y^5*Z^9**	Correct

Comment:

Remember to treat the other variables as constants.
The derivative is:

$$\frac{\partial F}{\partial X} = 140\, X^2\, Y^6\, Z^3 + 36\, Y^5\, Z^9$$

Question 7: *Score 1/1*

Your response	Correct response	
If the production function is $3\left(0.9\, L^5 + 0.1\, K^5\right)^{\frac{1}{5}}$, What is the marginal product of labour (L)? MPL= **(2.7*(((0.9*((L)^5.0))+(0.1*((K)^5.0)))^(-4/5)))*((L)^4.0)** (33%) What is the marginal product of capital (K)? MPK= **(0.3*(((0.9*((L)^5.0))+(0.1*((K)^5.0)))^(-4/5)))*((K)^4.0)** (33%) What is the marginal rate of technical substitution for labour with respect to capital ($MRTS_{LK}$)? $MRTS_{LK}$= **.1*K^4.0/L^4.0** (33%)	If the production function is $3\left(0.9\, L^5 + 0.1\, K^5\right)^{\frac{1}{5}}$, What is the marginal product of labour (L)? MPL= **(2.7*(((0.9*((L)^5.0))+(0.1*((K)^5.0)))^(-4/5)))*((L)^4.0)** What is the marginal product of capital (K)? MPK= **(0.3*(((0.9*((L)^5.0))+(0.1*((K)^5.0)))^(-4/5)))*((K)^4.0)** What is the marginal rate of technical substitution for labour with respect to capital ($MRTS_{LK}$)? $MRTS_{LK}$= **.1*K^4.0/L^4.0**	Correct

Comment:

The MPL is $2.7\left(0.9\, L^5 + 0.1\, K^5\right)^{-\frac{4}{5}} L^4$.

The MPK is $0.3\left(0.9\, L^5 + 0.1\, K^5\right)^{-\frac{4}{5}} K^4$.

The $MRTS_{LK}$ is $\frac{MPK}{MPL} = 0.1\, \frac{K^4}{L^4}$.

Question 8: *Score 1/1*

Your response	Correct response	
Differentiate $X+\frac{11}{4}XY+\frac{1}{8}Y$ with respect to Y. **11/4*X+1/8** (100%)	Differentiate $X+\frac{11}{4}XY+\frac{1}{8}Y$ with respect to Y. **11/4*X+1/8**	Correct

Comment:

Remember to treat X as a constant.

Question 9: *Score 1/1*

Your response	Correct response	
If the production function is $2\left(0.5L^4+0.5K^4\right)^{\frac{1}{4}}$, what is the marginal product of capital (K)? MPK= **((2.0*(((0.5*((L)^4.0))+(0.5*((K)^4.0)))^(-1.0*(3/4))))*0.5)*((K)^3.0)** (100%)	If the production function is $2\left(0.5L^4+0.5K^4\right)^{\frac{1}{4}}$, what is the marginal product of capital (K)? MPK= **((2.0*(((0.5*((L)^4.0))+(0.5*((K)^4.0)))^(-1.0*(3/4))))*0.5)*((K)^3.0)**	Correct

Comment:

The answer is MPK= $\left(0.5L^4+0.5K^4\right)^{-\frac{3}{4}}K^3$.

Question 10: *Score 1/1*

Your response	Correct response	
If the production function is $7L^{\frac{1}{7}}K^{\frac{1}{4}}$, what is the marginal product of capital (K)? MPK= **((7/4)*((L)^(1/7)))*((K)^(-3/4))** (100%)	If the production function is $7L^{\frac{1}{7}}K^{\frac{1}{4}}$, what is the marginal product of capital (K)? MPK= **((7/4)*((L)^(1/7)))*((K)^(-3/4))**	Correct

Comment:

The answer is MPK= $\frac{7}{4}L^{\frac{1}{7}}K^{-\frac{3}{4}}$.

Assignment 8: Multi-variable Optimization

Question 1: *Score 1/1*

Your response

A consumer wants to maximize her utility given by $U(x,y)=x^2y^2$ by choosing amounts of x and y to purchase. The price of good x is $P1$, the price of good $P2$ is , and the consumer has a total of M to spend.

Form the Lagrangean that best depicts this situation. Use J to denote the Lagrange multiplier, since λ is not available in the answer field. **x^2*y^2+J*(M-P1*x-P2*y)** (100%)

Correct

Comment:

The consumer wants to maximize utility subject to the budget constraint. The utility is given by $x^2\,y^2$, and the budget constraint is $p1\;x+p2\;y=M$. Therefore, we form the Lagrangean by taking the utility and adding to J (the Lagrange multiplier) times the budget constraint (rearranged so that it is all on the right hand side of the equation), to get $x^2\,y^2+J\,(M-P1\;x-P2\;y)$.

Question 2: *Score 1/1*

Your response

Consider the function $-27X^2 + 456X + 30XY - 12XZ - 3536 - 344Y - 256Z - 13Y^2 - 22Z^2$.
To optimize this function we find the first order conditions for X, Y and Z. Please give the first order conditions in the order X, Y and lastly Z.

-54*X+456+30*Y-12*Z (10%) =0

30*X-344-26*Y (10%) =0

-12*X-256-44*Z (10%) =0.

Solving, we find $X=$ **8** (10%) , $Y=$ **-4** (10%), and $Z=$ **-8** (10%).

Find the Hessian for this function. $\begin{bmatrix} -54 & 30 & -12 \\ 30 & -26 & 0 \\ -12 & 0 & -44 \end{bmatrix}$ (10%) .

The Hessian is **Negative definite** (10%) , and this is a **sufficient** (10%) condition for **a maximum** (10%) .

Correct

Comment:

The first order conditions are found by differentiating the function with respect to X first, then with respect to Y, and finally with respect to Z, to find:

$-54X + 456 + 30Y - 12Z$ =0,

$30X - 344 - 26Y$=0,

$-12X - 256 - 44Z$ =0.

These equations can be solved using various methods such as Gauss-Jordan, Cramer's rule, or row operations to reach the row echelon form.

The solutions are $X=$ 8, $Y=$ -4, and $Z=$ -8.

The Hessian is found by differentiating the first order conditions again, and arranging them in a matrix.

$$\begin{pmatrix} -54 & 30 & -12 \\ 30 & -26 & 0 \\ -12 & 0 & -44 \end{pmatrix}$$

To determine the nature of the Hessian we look at the principal minors, the determinants of the whole matrix, of the matrix formed by dropping the last row and column, and the top left entry.

The latter is -54 , the 2x2 determinant is det $\begin{pmatrix} -54 & 30 \\ 30 & -26 \end{pmatrix}$

and finally the determinant of the whole matrix is det $\begin{pmatrix} -54 & 30 & -12 \\ 30 & -26 & 0 \\ -12 & 0 & -44 \end{pmatrix}$.

You can verify that these are alternating in sign starting with negative, which means the that Hessian is negative definite, and thus the solution we found is a maximum.

Question 3: *Score 1/1*

Your response

A market for good X has demand $D_X = A - r \cdot p$ and supply $S_X = B + s \cdot p$, where p is the price.

The government wants to generate some revenue by introducing an excise tax t per unit sold. Give an expression for market clearing under the tax, by setting excess demand equal to zero. Naturally, with an excise tax, the price paid by the consumers will be different from the price received by the producers. Let p denote the price received by producers.

Excess Demand = **((A)-((r)*((p)+(t))))-((B)+((s)*(p)))** (25%) =0.

Solve this expression for the equilibrium price producers receive under the excise tax t. Equilibrium Producer Price= **(((A)-(B))-((t)*(r)))/((r)+(s))** (25%) .

Substitute this into the market demand to find the quantity under the excise tax t. Equilibrium Quantity= **(A)-((r)*(((((A)-(B))-((t)*(r)))/((r)+(s)))+(t)))** (25%)

Find an expression for the government revenue from this tax. Revenue = **(t)*((A)-((r)*(((((A)-(B))-((t)*(r)))/((r)+(s)))+(t))))** (25%) .

Correct

Comment:

Excess demand is demand minus supply. Since p denotes the price producers receive, the price paid by the consumers is $p + t$, which is what is used for the consumers' price in the demand. So, Supply is $B + s\,p$ and Demand is $A - r\,(p + t)$. To find excess demand, subtract supply from demand. $(A - r\,(p + t)) - (B + s\,p)$ Now solve for the price where excess demand is zero. $\dfrac{(A - B) - t\,r}{r + s}$

To find the equilibrium quantity, remember the above price is the producers' price. Since the question asks to use the demand to find equilibrium quantity, t must be added to the price, and then it is substituted into demand. $A - r\left(\dfrac{(A - B) - t\,r}{r + s} + t\right)$

(If instead the question had asked to use the supply curve, then the above price could be directly substituted, since it is the producers' price. $B + r\,\dfrac{(A - B) - t\,r}{r + s}$. Be sure to verify for yourself that this quantity is the same as the one found using the demand curve.)

Tax revenue is found by multiplying the equilibrium quantity by t. $t\left(A - r\left(\dfrac{(A - B) - t\,r}{r + s} + t\right)\right)$

Question 4: *Score 1/1*

Your response

Given the following function:

$$F(X, Y) = -65 + 85X + 66Y - 43X^2 + 98XY - 12Y^2$$

What are the stationary values (critical values)? (Enter your answers to at least 2 decimal places.)

$Xcrit =$ **-1.13** (50%)

$Ycrit =$ **-1.86** (50%)

Correct

Comment:

The critical values can be found by:
(1) Finding the derivate by X.
(2) Finding the derivate by Y.
(3) Setting both equations equal to zero and solving for X and Y.

Question 5: *Score 1/1*

Your response

Find the second order partial derivatives for the following function:

$F(X, Y) = 3X^5 Y^4 + 6Y^5 - 6X^2$

Second partial with respect to X:

$\frac{\partial^2 F}{\partial X^2}$ = **60*X^3*Y^4-12** (25%)

Second partial with respect to Y:

$\frac{\partial^2 F}{\partial Y^2}$ = **36*X^5*Y^2+120*Y^3** (25%)

Cross-partial with respect to X, Y:

$\frac{\partial^2 F}{\partial YX}$ = **60*X^4*Y^3** (25%)

Cross-partial with respect to Y, X:

$\frac{\partial^2 F}{\partial YX}$ = **60*X^4*Y^3** (25%)

Correct

Comment:

Remember to treat the other variable in each case as a constant.

Second partial with respect to X is:

$$\frac{\partial^2 F}{\partial X^2} = 60X^3 Y^4 - 12$$

Second partial with respect to Y is:

$$\frac{\partial^2 F}{\partial Y^2} = 36X^5 Y^2 + 120Y^3$$

Cross-partial with respect to X,Y is:

$$\frac{\partial^2 F}{\partial XY} = 60X^4 Y^3$$

Cross-partial with respect to Y,X is:

$$\frac{\partial^2 F}{\partial XY} = 60X^4 Y^3$$

Question 6: *Score 1/1*

Your response

A regulated monopoly faces as demand $\mathrm{D} = A - mP$, where P is the market price and produces according to the technology $q = K^a L^b$. Give an expression for the firm's revenue as a function of capital, K, and labor, L.

Revenue = **(A-(1/m)*(K^a)*(L^b))*((K^a)*(L^b))** (20%)

If we use w to denote the wage rate and r to denote the price of capital, the firm's profit without regulation is **(A-(1/m)*(K^a)*(L^b))*((K^a)*(L^b))-(r*K)-(w*L)** (20%) .

The regulatory agency allows the regulated firm to earn a return of s per unit of capital. To find the total return on capital we take the revenue minus the total amount paid to labour. This is the total return on capital. The regulation requires that the per unit return on capital can be at most s . This is a constraint that the monopoly faces. Write down this constraint as an equality. **((A-(1/m)*(K^a)*(L^b))*((K^a)*(L^b))-(w*L))/K** (20%) =s.

The monopoly's constrained problem can be written using a Lagrange multiplier as

$Max_{K,L}$ **(A-(1/m)*(K^a)*(L^b))*((K^a)*(L^b))-(r*K)-(w*L)** (20%) + λ **s-((A-(1/m)*(K^a)*(L^b))*((K^a)*(L^b))-(w*L))/K** (20%) .

Correct

Comment:

The revenue is found by first inverting the demand to find $P = A - \frac{q}{m}$. Then we find revenue by multiplying price times quantity, $\left(A - \frac{K^a L^b}{m}\right) K^a L^b$. Profit is found by subtracting from revenue the cost of the monopoly of the labor and capital, $\left(A - \frac{K^a L^b}{m}\right) K^a L^b - r K - w L$.

For rate of return regulation we first find the total return on capital, that is the revenue less the total cost of labour. Then we divide that amount by the amount of capital used. This gives us the return per unit of capital, $\frac{\left(A - \frac{K^a L^b}{m}\right) K^a L^b - w L}{K}$.

We combine everything to find the Lagrangean. $\left(A - \frac{K^a L^b}{m}\right) K^a L^b - r K - w L + \lambda \left(s - \frac{\left(A - \frac{K^a L^b}{m}\right) K^a L^b - w L}{K}\right)$

Question 7: *Score 1/1*

Your response

A firm receives a total revenue of $50 \cdot q - 2 \cdot q^2$, and its total costs are $q^2 + 10 \cdot q + 50$. Give an expression for its profit. Profit= **50*q-2*q^2-q^2-10*q-50** (33%) .

If the government levied an excise tax of t dollars per unit sold, now what would its profit be?

Profit = **50*q-2*q^2-q^2-10*q-50-t*q** (33%) .

Instead if the government imposed a t % profit tax, what would the firm's profits be?

Profit = **(1-t)*(50*q-2*q^2-q^2-10*q-50)** (33%)

Correct

Comment:

The firm's profit is $40\,q - 3\,q^2 - 50$. If we impose and excise tax, the profit now becomes $40\,q - 3\,q^2 - 50 - t\,q$. Instead if there is a profit tax the firm's after tax profit is $(1 - t)\left(40\,q - 3\,q^2 - 50\right)$.

Question 8: *Score 1/1*

Your response

Given the following function:

$F(X, Y) = -40 + 78\,X + 99\,Y + 93\,X^2 - 53\,X\,Y + 41\,Y^2$

What are the stationary values (critical values)? (Enter your answers to at least 2 decimal places.)

$Xcrit =$ **-0.94** (33%)

$Ycrit$ = **-1.81** (33%)

Is the critical point a maximum, minimum or is it indeterminate?

$Xcrit$ is a **min** (33%)

Correct

Comment:

The critical values can be found by:
(1) Finding the derivate by X.
(2) Finding the derivate by Y.
(3) Setting both equations equal to zero and solving for X and Y.

The nature of the critical values can be found by:
(1) Finding the Hessian matrix.
(2) Finding the eigenvalues of the Hessian matrix.
(3) If they are both positive, it is a minimum. If they are both negative, it is a maximum.

Question 9: *Score 1/1*

Your response

Consider the function $288 - 82X - 18Y + 66Z + 12X^2 - 4XY - 2XZ - 6YZ + 6Z^2$. To optimize this function we find the first order conditions for X, Y and Z. Please give the first order conditions in the order X, Y and lastly Z.

24*X-82-4*Y-2*Z (10%) =0

-4*X-18-6*Z (10%) =0

-2*X+66-6*Y+12*Z (10%) =0.

Solving, we find $X=$ **3** (10%) , $Y=$ **0** (10%), and $Z=$ **-5** (10%).

Correct

Find the Hessian for this function. $\begin{bmatrix} 24 & -4 & -2 \\ -4 & 0 & -6 \\ -2 & -6 & 12 \end{bmatrix}$ (10%) .

The Hessian is **Indefinite** (10%) , and this is a **neither necessary nor sufficient** (10%) condition for **an inconclusive result** (10%) .

Comment:

The first order conditions are found by differentiating the function with respect to X first, then with respect to Y, and finally with respect to Z, to find:

$24X - 82 - 4Y - 2Z$ =0,

$-4X - 18 - 6Z$ =0,

$-2X + 66 - 6Y + 12Z$ =0.

These equations can be solved using various methods such as Gauss-Jordan, Cramer's rule, or row operations to reach the row echelon form.

The solutions are $X=$ 3, $Y=$ 0, and $Z=$ -5.

The Hessian is found by differentiating the first order conditions again, and arranging them in a matrix.

$$\begin{pmatrix} 24 & -4 & -2 \\ -4 & 0 & -6 \\ -2 & -6 & 12 \end{pmatrix}$$

To determine the nature of the Hessian we look at the principal minors, the determinants of the whole matrix, of the matrix formed by dropping the last row and column, and the top left entry.

The latter is 24, the 2x2 determinant is det $\begin{pmatrix} 24 & -4 \\ -4 & 0 \end{pmatrix}$

and finally the determinant of the whole matrix is det $\begin{pmatrix} 24 & -4 & -2 \\ -4 & 0 & -6 \\ -2 & -6 & 12 \end{pmatrix}$.

You can verify that these are neither all positive nor alternating in sign starting with negative, which means the that Hessian is indefinite, and thus the solution we found is neither a maximum nor a minimum.

Question 10: *Score 1/1*

Your answer:

Your response
In an optimization problem, the **second** **order** **conditions** indicate whether the solution to the first order conditions is a maximum, and minimum or neither.

Correct

Assignment 9: Multi-variable Optimization and the Hessian

Question 1: *Score 1/1*

Your response

Consider the function $-36 + 50X - 62Y + 86X^2 + 61XY + 14X^3$, which is being optimized with the constraint $-8X - 7Y = 0$. Find the Bordered Hessian matrix associated with this problem.

(Enter it in the order X ,Y ,lambda.) $\begin{bmatrix} 172 + 84 \cdot X & 61 & -8 \\ 61 & 0 & -7 \\ -8 & -7 & 0 \end{bmatrix}$ (33%).

Correct

At $[X, Y] = \begin{pmatrix} 1 & 1 \end{pmatrix}$, we evaluate the matrix to find $\begin{bmatrix} 256 & 61 & -8 \\ 61 & 0 & -7 \\ -8 & -7 & 0 \end{bmatrix}$ (33%) .

If the first order conditions held at this point the Hessian would **indicate a minimum** (33%).

Comment:

To find the bordered Hessian, we first construct the Lagrangean, $-36 + 50X - 62Y + 86X^2 + 61XY + 14X^3 + \lambda(-8X - 7Y)$.

The Hessian is found by twice differentiating the Lagrangean, and arranging the second derivatives in a matrix,

$$\begin{pmatrix} 172 + 84X & 61 & -8 \\ 61 & 0 & -7 \\ -8 & -7 & 0 \end{pmatrix}.$$

Then at the point $(X, Y) = \begin{pmatrix} 1 & 1 \end{pmatrix}$, the evaluated Hessian is $\begin{pmatrix} 256 & 61 & -8 \\ 61 & 0 & -7 \\ -8 & -7 & 0 \end{pmatrix}$. (If the first order conditions held at this point), a sufficient condition for the point $(X, Y) = \begin{pmatrix} 1 & 1 \end{pmatrix}$ to be a maximum is that the determinant if the evaluated Hessian matrix is positive; for a minimum it is negative; and with a zero determinant it is inconclusive.

In this case, $det \begin{pmatrix} 256 & 61 & -8 \\ 61 & 0 & -7 \\ -8 & -7 & 0 \end{pmatrix} = $ -5,712, which means that the Hessian indicates a minimum.

Question 2: *Score 1/1*

Your response

A firm produces using the technology $K^{.3} L^{.4}$. It wants to minimize its cost of producing a quantity, q. The price of capital is 3 and the wage is 5. Give the Lagrangean associated with this problem:
3.0*K+5.0*L (11%) $+ \lambda$ **q-K^.3*L^.4** (11%) .

Letting "J" denote the Lagrange multiplier, give the first order conditions starting with the one for K and ending with the one for J.
3.0-.3*J/K^.7*L^.4 (11%) = **0** (11%) ,
5.0-.4*J*K^.3/L^.6 (11%) = **0** (11%) , and
q-K^.3*L^.4 (11%) = **0** (11%) .

Correct

The Hessian is $\begin{bmatrix} \frac{0.21 \cdot J \cdot L^{0.4}}{K^{1.7}} & -\frac{0.12 \cdot J}{K^{0.7} L^{0.6}} & -\frac{0.3 \cdot L^{0.4}}{K^{0.7}} \\ -\frac{0.12 \cdot J}{K^{0.7} L^{0.6}} & \frac{0.24 \cdot J \cdot K^{0.3}}{L^{1.6}} & -\frac{0.4 \cdot K^{0.3}}{L^{0.6}} \\ -\frac{0.3 \cdot L^{0.4}}{K^{0.7}} & -\frac{0.4 \cdot K^{0.3}}{L^{0.6}} & 0 \end{bmatrix}$ (11%) . (Enter it in the order K, L, J.)

Comment:

The firm wants to minimize cost subject to a production constraint. So, the Lagrangean is 5*L+3*K+$\lambda($ q - $K^{.3} L^{.4}$ $)$. To find the First order conditions we differentiate the Lagrangean with respect to K, L and λ. Letting J denote λ, we find

$$3.0 - .3 \frac{1.\ J\ L^{.4}}{1.\ K^{.7}} = 0$$

$$5.0 - .4 \frac{1.\ J\ K^{.3}}{1.\ L^{.6}} = 0$$

$$q - K^{.3} L^{.4} = 0$$

The Hessian for this problem is $\begin{pmatrix} .21 \frac{1.\ J\ L^{.4}}{1.\ K^{1.7}} & -.12 \frac{1.\ J}{K^{.7} L^{.6}} & -.3 \frac{1.\ L^{.4}}{1.\ K^{.7}} \\ -.12 \frac{1.\ J}{K^{.7} L^{.6}} & .24 \frac{1.\ J\ K^{.3}}{1.\ L^{1.6}} & -.4 \frac{1.\ K^{.3}}{1.\ L^{.6}} \\ -.3 \frac{1.\ L^{.4}}{1.\ K^{.7}} & -.4 \frac{1.\ K^{.3}}{1.\ L^{.6}} & 0 \end{pmatrix}$

Question 3: *Score 1/1*

Your response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = \frac{1}{3}\ln(X) + \frac{1}{4}\ln(Y)$$

The price of X is Px=3.02, and the price of Y is Py=1.76. The individual has 37 dollars.

How much X and Y should this individual buy?

(Round answers to the nearest whole number.)

X= **7** (50%)

Y= **9** (50%)

Correct

Comment:

The Lagrangean is $\mathcal{L} = \frac{1}{3}\ln(X) + \frac{1}{4}\ln(Y) + \lambda$ [37-3.02*X-1.76*Y].

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial X} : \frac{\frac{1}{3}}{X} - 3.02\,L = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} : \frac{\frac{1}{4}}{Y} - 1.76\,L = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : (37 - 3.02\,X) - 1.76\,Y = 0$$

Optimal X and Y are:

X=7

Y=9

Question 4: *Score 1/1*

Your response

A firm has the following production function:

$$Q(K,L) = K^{\frac{1}{3}} L^{\frac{2}{3}}$$

The price of K is P_K =3, and the price of L is P_L =2. The firm wishes to produce 16 units for the least cost possible.

What is the Lagrangean for this problem?

min (5%) $\mathcal{L}$ = **3*K+2*L** (5%) + λ [**16-K^(1/3)*L^(2/3)** (5%)]

What are the first order conditions? **Note: use m for λ (because in this question L means labour).**

$\frac{\partial \mathcal{L}}{\partial K}$: **3-1/3*m/K^(2/3)*L^(2/3)** (5%) = **0** (5%)

$\frac{\partial \mathcal{L}}{\partial L}$: **2-2/3*m*K^(1/3)/L^(1/3)** (5%) = **0** (5%)

$\frac{\partial \mathcal{L}}{\partial \lambda}$: **16-K^(1/3)*L^(2/3)** (5%) = **0** (5%)

Correct

How much K and L should the firm use?

(Round answers to the nearest two decimal places. For example, 1.6666 becomes 1.67.)

K= **7.69** (26%)

L= **23.08** (26%)

Comment:

The Lagrangean is $\mathcal{L}$ = 3*K+2*L + λ 16-[$K^{\frac{1}{3}} L^{\frac{2}{3}}$].

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial K} : 3 - \frac{1}{3} \frac{m}{K^{\frac{2}{3}}} L^{\frac{2}{3}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial L} : 2 - \frac{2}{3} m \frac{K^{\frac{1}{3}}}{L^{\frac{1}{3}}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : 16 - K^{\frac{1}{3}} L^{\frac{2}{3}} = 0$$

Optimal K and L are:

X=7.69

Y=23.08

Question 5: *Score 1/1*

Your response

A firm produces and sells two goods: X and Y. Both goods sell for a price of $1 each.

The production function for good X is $X = L_X$.

The production function for good Y is $Y = K^{\frac{1}{2}} L_Y$.

The firm divides its labour between the two products: $L = L_X + L_Y$ and gets its labour for free, but can only use 10 units of it or less. $L \leq 10$.

The price of capital is $p_K = 3$.

What is the firm's cost function if it puts all its labour into X? (Assuming it does not try to use more than 10 units of labour.)

$C(X) =$ **0** (8%)

What is the firm's profit if it puts all its labour into X?

$\pi(L_X = 10) =$ **10** (8%)

What is the firm's cost function if it puts all its labour into Y?

$C(Y) =$ **(3/100)*Y^2** (8%)

What is the firm's profit function if it puts all its labour into Y?

$\pi(L_Y = 10) =$ **Y-3/100*Y^2** (8%)

How much Y should the firm produce if it puts all its labour into Y?

$Y =$ **50/3** (8%)

What is the firm's profit if it puts all its labour into Y?

$\pi(L_Y = 10) =$ **25/3** (8%)

Given the above results, how much of each input should the firm use, and how much of each output should it produce?

L_X = **10** (8%)

L_Y = **0** (8%)

$K =$ **0** (8%)

X = **10** (8%)

$Y =$ **0** (8%)

How much profit does the firm make?

$\pi =$ **10** (8%)

Correct

Comment:

This question has a corner solution. If the price of capital is low enough, it is best to put all the firm's labour into producing good Y. Otherwise, it is best to put all the firm's labour into producing good X.

In this case, the price of capital is 3. Therefore, it is best to choose:

$L_X =$ 10

$L_Y =$ 0

$K =$ 0

$X =$ 10

$Y =$ 0

Which gives a profit of:

$\pi =$ 10

Question 6: *Score 1/1*

Your response

Consider the function $-70 + 18X - 35XY + 31X^2Y - 33XY^2 + 65Y^3$. Find the Hessian matrix associated with this function. $\begin{bmatrix} 62Y & -35 + 62X - 66Y \\ -35 + 62X - 66Y & -66X + 390Y \end{bmatrix}$ (33%).

Correct

At $[X, Y] = \begin{pmatrix} 0 & 0 \end{pmatrix}$ the Hessian is **indefinite** (33%); if the first order conditions held at this point it would **be inconclusive** (33%).

Comment:

The Hessian is found by twice differentiating the function $-70 + 18X - 35XY + 31X^2Y - 33XY^2 + 65Y^3$, and arranging the second derivatives in a matrix. $\begin{pmatrix} 62Y & -35 + 62X - 66Y \\ -35 + 62X - 66Y & -66X + 390Y \end{pmatrix}$

Then we evaluate the matrix at $(X, Y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$ to find $\begin{pmatrix} 0 & -35 \\ -35 & 0 \end{pmatrix}$.

We know that if $(X, Y) = \begin{pmatrix} 0 & 0 \end{pmatrix}$ satisfied the first order conditions, then for this to be a minimum a sufficient condition is that the Hessian is positive semi-definite. This requires that the principal minors are all positive. Recall that the principal minors are determinants of matrices formed from the Hessian by starting with the 1x1 matrix containing the upper left most entry. Then we look at the 2x2 by adding one row from below and one column from the right. We keep going until we finally have the entire Hessian matrix.

For a maximum the principal minors are alternating in sign starting with negative.

For any other situation the Hessian is inconclusive and the point may be a maximum a minimum or neither.

In this instance since the Hessian is 2x2 we look at the upper left hand entry and then the determinant of the whole matrix. Looking at the upper left entry 0 and $det \begin{pmatrix} 0 & -35 \\ -35 & 0 \end{pmatrix}$ = -1,225, we find that the Hessian is indefinite, which is inconclusive.

Question 7: *Score 1/1*

Your response

A firm produces and sells two goods: X and Y. Both goods sell for a price of $1 each.

The production function for good X is $X = L_X$

The production function for good Y is $Y = L_Y^{\frac{1}{2}} K$

The firm divides its labour between the two products: $L = L_X + L_Y$ and gets its labour for free, but can only use 10 units of it or less $L \leq L_X + L_Y$.

The price of capital is $P_K = 4$.

How much labour should the firm use to produce good X ?

$L_X =$ **10** (17%)

Correct

How much labour should the firm use to produce good Y?

$L_Y =$ **0** (17%)

How much capital should the firm use?

$K =$ **0** (17%)

How much of good X should the firm produce?

$X =$ **10** (17%)

How much of good Y should the firm produce?

$Y =$ **0** (17%)

How much profit does the firm make?

$\pi =$ **10** (17%)

Comment:

This question has a corner solution. If the price of capital is low enough, it is best to put all the firm's labour into producing good Y. Otherwise, it is best to put all the firm's labour into producing good X.

In this case, the price of capital is 4. Therefore, it is best to choose:

$L_X =$ 10

$L_Y =$ 0

$K =$ 0

$X =$ 10

$Y =$ 0

Which gives a profit of:

$\pi =$ 10

Question 8: *Score 1/1*

Your response

Consider the function $17X - 24XY - 57XZ + 94X^2Z - 77YZ^2 + 36Z^3$. Find the Hessian matrix associated with this function. $\begin{bmatrix} 188Z & -24 & -57+188X \\ -24 & 0 & -154Z \\ -57+188X & -154Z & -154Y+216Z \end{bmatrix}$ (33%)

Correct

At $\begin{bmatrix} X, & Y, & Z \end{bmatrix} = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$, the Hessian is **indefinite** (33%); if the first order conditions held at this point it would **be inconclusive** (33%).

Comment:

The Hessian is found by twice differentiating the function $17X - 24XY - 57XZ + 94X^2Z - 77YZ^2 + 36Z^3$, and arranging the second derivatives in a matrix $\begin{pmatrix} 188Z & -24 & -57+188X \\ -24 & 0 & -154Z \\ -57+188X & -154Z & -154Y+216Z \end{pmatrix}$.

Then we evaluate the matrix at $(X, Y, Z) = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$ to find $\begin{pmatrix} 0 & -24 & -245 \\ -24 & 0 & 0 \\ -245 & 0 & -154 \end{pmatrix}$.

We know that if $(X, Y, Z) = \begin{pmatrix} -1 & 1 & 0 \end{pmatrix}$ satisfied the first order conditions, then for this to be a minimum a sufficient condition is that the Hessian is positive semi-definite. This requires that the principal minors are all positive. Recall that the principal minors are determinants of matrices formed from the Hessian by starting with the 1x1 matrix containing the upper left most entry. Then we look at the 2x2 by adding one row from below and one column from the right. We keep going until we finally have the entire Hessian matrix.

For a maximum the principal minors are alternating in sign starting with negative.

For any other situation the Hessian is inconclusive and the point may be a maximum a minimum or neither.

In this instance since the Hessian is 3x3 we look at the upper left hand entry, then the determinant of the upper left 2x2 matrix, and finally the determinant of the whole matrix. Looking at the upper left entry (R)*1.0, the determinant of the upper left 2x2 matrix, $det($ (R)*2.0 $) =$ -576, and the whole matrix, $det\left(\begin{pmatrix} 0 & -24 & -245 \\ -24 & 0 & 0 \\ -245 & 0 & -154 \end{pmatrix}\right) =$ 88,704, we find that the Hessian indefinite, which is inconclusive.

Question 9: *Score 1/1*

Your response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = 5\left(0.5\,X^{-0.75} + 0.5\,Y^{-0.75}\right)^{-1.33}$$

The price of X is Px=2, and the price of Y is Py=4. The individual has 18 dollars.

How much X and Y should this individual buy?

(Round answers to the nearest two decimals. For example, 1.666 rounds to 1.67.)

X= **3.84** (50%)

Y= **2.58** (50%)

Correct

Comment:

The Lagrangean is $\mathcal{L} = 5\left(0.5\,X^{-0.75} + 0.5\,Y^{-0.75}\right)^{-1.33} + \lambda$ [18-(P)*(x)*X-(P)*(y)*Y].

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial X} : \frac{5}{2} \frac{1}{\left(\frac{1}{2X^{\frac{3}{4}}} + \frac{1}{2Y^{\frac{3}{4}}}\right)^{\frac{7}{3}} X^{\frac{7}{4}}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} : \frac{5}{2} \frac{1}{\left(\frac{1}{2X^{\frac{3}{4}}} + \frac{1}{2Y^{\frac{3}{4}}}\right)^{\frac{7}{3}} Y^{\frac{7}{4}}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : 18 - 2X - 4Y = 0$$

Optimal X and Y are:

X=3.84

Y=2.58

Question 10: *Score 1/1*

Your response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = \frac{1}{2}\ln(X) + \frac{1}{3}\ln(Y)$$

The price of X is Px=3.1, and the price of Y is Py=2.07. The individual has 31 dollars.

What is the Lagrangean for this problem?

$\mathcal{L}$ = **1/2*ln(X)+1/3*ln(Y)** (6%) + λ [**31-3.1*X-2.07*Y** (6%)]

What are the first order conditions? Use L for λ.

$\frac{\partial \mathcal{L}}{\partial X}$: **1/2/X-3.1*L** (6%) = **0** (6%)

$\frac{\partial \mathcal{L}}{\partial Y}$: **1/3/Y-2.07*L** (6%) = **0** (6%)

$\frac{\partial \mathcal{L}}{\partial \lambda}$: **31-3.1*X-2.07*Y** (6%) = **0** (6%)

How much X and Y should this individual buy?

(Round answers to the nearest whole number.)

X= **6** (28%)

Y= **6** (28%)

Correct

Comment:

The Lagrangean is $\mathcal{L} = \frac{1}{2}\ln(X) + \frac{1}{3}\ln(Y)$ + λ[31-3.1*X-2.07*Y].

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial X} : \frac{\frac{1}{2}}{X} - 3.1\,L = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} : \frac{\frac{1}{3}}{Y} - 2.07\,L = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : (31 - 3.1\,X) - 2.07\,Y = 0$$

Optimal X and Y are:

X=6

Y=6

Assignment 10

Question 1: *Score 2/2*

Your response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = \frac{1}{4}\ln(X) + \frac{1}{3}\ln(Y)$$

The price of X is Px=2.45, and the price of Y is Py=4.57. The individual has 40 dollars.

What is the Lagrangean for this problem?

$\mathcal{L}$ = **.25*ln(X)+.3333333333333333*ln(Y)** (6%) + λ [**40-2.45*X-4.57*Y** (6%)]

What are the first order conditions? Type L for λ.

$\frac{\partial\mathcal{L}}{\partial X}$: **1/4/X-2.45*L** (6%) = **0** (6%)

$\frac{\partial\mathcal{L}}{\partial Y}$: **1/3/Y-4.57*L** (6%) = **0** (6%)

$\frac{\partial\mathcal{L}}{\partial \lambda}$: **40-2.45*X-4.57*Y** (6%) = **0** (6%)

How much X and Y should this individual buy? (Round answers to the nearest whole number.)

X= **7** (28%)

Y= **5** (28%)

Correct response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = \frac{1}{4}\ln(X) + \frac{1}{3}\ln(Y)$$

The price of X is Px=2.45, and the price of Y is Py=4.57. The individual has 40 dollars.

What is the Lagrangean for this problem?

$\mathcal{L}$ = **.25*ln(X)+.3333333333333333*ln(Y)** + λ [**40-2.45*X-4.57*Y**]

What are the first order conditions? Type L for λ.

$\frac{\partial\mathcal{L}}{\partial X}$: **1/4/X-2.45*L** = **0**

$\frac{\partial\mathcal{L}}{\partial Y}$: **1/3/Y-4.57*L** = **0**

$\frac{\partial\mathcal{L}}{\partial \lambda}$: **40-2.45*X-4.57*Y** = **0**

How much X and Y should this individual buy? (Round answers to the nearest whole number.)

X= **7**

Y= **5**

Correct

Comment:

The Lagrangean is $\mathcal{L} = \frac{1}{4}\ln(X) + \frac{1}{3}\ln(Y) + \lambda$[40-2.45*X-4.57*Y].The first order conditions are:

$$\frac{\partial\mathcal{L}}{\partial X} : \frac{\frac{1}{4}}{X} - 2.45\,L = 0$$

$$\frac{\partial\mathcal{L}}{\partial Y} : \frac{\frac{1}{3}}{Y} - 4.57\,L = 0$$

$$\frac{\partial\mathcal{L}}{\partial \lambda} : (40 - 2.45\,X) - 4.57\,Y = 0$$

Optimal X and Y are:

X=7, Y=5

Question 2: *Score 2/2*

Your response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = X^{\frac{1}{4}} Y^{\frac{1}{4}}$$

The price of X is Px=2.44, and the price of Y is Py=1.95. The individual has 39 dollars.

What is the Lagrangean for this problem?

$\mathcal{L}$ = **X^(1/4)*Y^(1/4)** (6%) + λ [**39-2.44*X-1.95*Y** (6%)]

What are the first order conditions? Type L for λ.

$\frac{\partial \mathcal{L}}{\partial X}$: **1/4/X^(3/4)*Y^(1/4)-2.44*L** (6%) = **0** (6%)

$\frac{\partial \mathcal{L}}{\partial Y}$: **1/4*X^(1/4)/Y^(3/4)-1.95*L** (6%) = **0** (6%)

$\frac{\partial \mathcal{L}}{\partial \lambda}$: **39-2.44*X-1.95*Y** (6%) = **0** (6%)

How much X and Y should this individual buy?

(Round answers to the nearest whole number.)

X= **8** (28%)

Y= **10** (28%)

Correct response

An individual values two goods, X and Y, according to the following utility function:

$$U(X, Y) = X^{\frac{1}{4}} Y^{\frac{1}{4}}$$

The price of X is Px=2.44, and the price of Y is Py=1.95. The individual has 39 dollars.

What is the Lagrangean for this problem?

$\mathcal{L}$ = **X^(1/4)*Y^(1/4)** + λ [**39-2.44*X-1.95*Y**]

What are the first order conditions? Type L for λ.

$\frac{\partial \mathcal{L}}{\partial X}$: **1/4/X^(3/4)*Y^(1/4)-2.44*L** = **0**

$\frac{\partial \mathcal{L}}{\partial Y}$: **1/4*X^(1/4)/Y^(3/4)-1.95*L** = **0**

$\frac{\partial \mathcal{L}}{\partial \lambda}$: **39-2.44*X-1.95*Y** = **0**

How much X and Y should this individual buy?

(Round answers to the nearest whole number.)

X= **8**

Y= **10**

Correct

Comment:

The Lagrangean is $\mathcal{L} = X^{\frac{1}{4}} Y^{\frac{1}{4}} + \lambda$[39-2.44*X-1.95*Y]. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial X} : \frac{\frac{1}{4}}{X^{\frac{3}{4}}} Y^{\frac{1}{4}} - 2.44\, L = 0$$

$$\frac{\partial \mathcal{L}}{\partial Y} : \frac{1}{4} \frac{X^{\frac{1}{4}}}{Y^{\frac{3}{4}}} - 1.95\, L = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} : (39 - 2.44\, X) - 1.95\, Y = 0$$

Optimal X and Y are:

X=8, Y=10

Question 3: *Score 2/2*

Your response	Correct response
Given the following function: $F(X, Y) =$ $-47 - 54\,X + 29\,Y - 10\,X^2 + 98\,X\,Y - 64\,Y^2$ Find the first order conditions for optimizing this function: $\frac{\partial F}{\partial X}$ = **-54-20*X+98*Y** (9%) = **0** (9%) $\frac{\partial F}{\partial Y}$ = **29+98*X-128*Y** (9%) = **0** (9%) Given the above first order conditions, what are the stationary points (critical values)? (Enter your answers to at least 2 decimal places.) $Xcrit =$ **0.58** (9%) $Ycrit$ = **0.67** (9%) What are the second derivatives? $\frac{\partial^2 F}{\partial X^2}$ = **-20** (9%) $\frac{\partial^2 F}{\partial Y^2}$ = **-128** (9%) $\frac{\partial^2 F}{\partial XY}$ = **98** (9%) $\frac{\partial^2 F}{\partial YX}$ = **98** (9%) Given the above second derivatives, is the critical point a maximum, minimum or is it indeterminate? $Xcrit$ is a **indeterminate** (9%)	Given the following function: $F(X, Y) =$ $-47 - 54\,X + 29\,Y - 10\,X^2 + 98\,X\,Y - 64\,Y^2$ Find the first order conditions for optimizing this function: $\frac{\partial F}{\partial X}$ = **-54-20*X+98*Y** = **0** $\frac{\partial F}{\partial Y}$ = **29+98*X-128*Y** = **0** Given the above first order conditions, what are the stationary points (critical values)? (Enter your answers to at least 2 decimal places.) $Xcrit =$ **0.58** $Ycrit$ = **0.67** What are the second derivatives? $\frac{\partial^2 F}{\partial X^2}$ = **-20** $\frac{\partial^2 F}{\partial Y^2}$ = **-128** $\frac{\partial^2 F}{\partial XY}$ = **98** $\frac{\partial^2 F}{\partial YX}$ = **98** Given the above second derivatives, is the critical point a maximum, minimum or is it indeterminate? $Xcrit$ is a **indeterminate**

Correct

Comment:

The critical values (stationary points) can be found by setting both derivatives equal to zero and solving for X and Y.

The nature of the critical values can be found by finding the eigenvalues of the Hessian matrix. If they are both positive, it is a minimum. If they are both negative, it is a maximum.

Question 4: *Score 2/2*

Your response	Correct response

An individual values two goods, X and Y, according to the following utility function:

$$U(X,Y) = X^{\frac{1}{3}} Y^{\frac{1}{4}}$$

The price of X is Px=3.27, and the price of Y is Py=1.9. The individual has 40 dollars.

If the first order derivatives are the following, (where L is lambda)

$$\frac{\partial \mathcal{L}}{\partial X} : \frac{\frac{1}{3}}{X^{\frac{2}{3}}} Y^{\frac{1}{4}} - 3.27\,L$$

$$\frac{\partial \mathcal{L}}{\partial Y} : \frac{1}{4} \frac{X^{\frac{1}{3}}}{Y^{\frac{3}{4}}} - 1.9\,L$$

$$\frac{\partial \mathcal{L}}{\partial L} : (40 - 3.27\,X) - 1.9\,Y$$

Find the bordered Hessian:

(To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.

Also, please make sure to use capital X and Y and be careful about order of operations when entering answers.)

H=

$$\begin{bmatrix} -\frac{2}{9} \cdot \frac{\sqrt[4]{Y}}{X^{\frac{5}{3}}} & \frac{1}{12 \cdot X^{\frac{2}{3}} \cdot Y^{\frac{3}{4}}} & -3.27 \\ \frac{1}{12 \cdot X^{\frac{2}{3}} \cdot Y^{\frac{3}{4}}} & -\frac{3}{16} \frac{\sqrt[3]{X}}{Y^{\frac{7}{4}}} & -1.9 \\ -3.27 & -1.9 & 0 \end{bmatrix}$$

(100%) (Enter in the order X, Y, lambda.)

An individual values two goods, X and Y, according to the following utility function:

$$U(X,Y) = X^{\frac{1}{3}} Y^{\frac{1}{4}}$$

The price of X is Px=3.27, and the price of Y is Py=1.9. The individual has 40 dollars.

If the first order derivatives are the following, (where L is lambda)

$$\frac{\partial \mathcal{L}}{\partial X} : \frac{\frac{1}{3}}{X^{\frac{2}{3}}} Y^{\frac{1}{4}} - 3.27\,L$$

$$\frac{\partial \mathcal{L}}{\partial Y} : \frac{1}{4} \frac{X^{\frac{1}{3}}}{Y^{\frac{3}{4}}} - 1.9\,L$$

$$\frac{\partial \mathcal{L}}{\partial L} : (40 - 3.27\,X) - 1.9\,Y$$

Find the bordered Hessian:

(To input your answer, right-click on the box below to bring up the symbols bar, select the button with a square made out of 9 smaller squares and select the appropriate size of matrix. If the correct size is not shown, select $n \times m$ and set the dimensions yourself.

Also, please make sure to use capital X and Y and be careful about order of operations when entering answers.)

H=

$$\begin{bmatrix} -\frac{2}{9} \cdot \frac{\sqrt[4]{Y}}{X^{\frac{5}{3}}} & \frac{1}{12 \cdot X^{\frac{2}{3}} \cdot Y^{\frac{3}{4}}} & -3.27 \\ \frac{1}{12 \cdot X^{\frac{2}{3}} \cdot Y^{\frac{3}{4}}} & -\frac{3}{16} \frac{\sqrt[3]{X}}{Y^{\frac{7}{4}}} & -1.9 \\ -3.27 & -1.9 & 0 \end{bmatrix}$$

(Enter in the order X, Y, lambda.)

Correct

Question 5: *Score 2/2*

Your response

A firm has a technology $Q = K^a L^b$. If the price of labour is denoted by w and the price of capital is v , and the firm wants to produce Q units of output, give the Lagrangean used in solving the firm's cost minimization problem. Since λ is not available in the solution field, please use "J" for the Lagrange multiplier.

Lagrangean $=$ **(w*L+v*K)+J*(Q-K^a*L^b)** (17%) .

The first order conditions are (starting with K and ending with the Lagrange multiplier J)

v-J*K^a*(a/K)*L^b (17%) $= 0$

w-J*K^a*L^b*(b/L) (17%) $= 0$

Q-K^a*L^b (17%) $= 0$

Solving these we find $K =$ **(a*w/(b*v))^(a/(a+b))*Q^(1/(a+b))** (17%) , and $L =$ **(b*v/a/w)^(b/(a+b))*Q^(1/(a+b))** (17%) .

Correct response

A firm has a technology $Q = K^a L^b$. If the price of labour is denoted by w and the price of capital is v , and the firm wants to produce Q units of output, give the Lagrangean used in solving the firm's cost minimization problem. Since λ is not available in the solution field, please use "J" for the Lagrange multiplier.

Lagrangean $=$ **(w*L+v*K)+J*(Q-K^a*L^b)** .

The first order conditions are (starting with K and ending with the Lagrange multiplier J)

v-J*K^a*(a/K)*L^b $= 0$

w-J*K^a*L^b*(b/L) $= 0$

Q-K^a*L^b $= 0$

Solving these we find $K =$ **(a*w/(b*v))^(a/(a+b))*Q^(1/(a+b))** , and $L =$ **(b*v/a/w)^(b/(a+b))*Q^(1/(a+b))** .

Correct

Comment:

To find the Lagrangean for the firm's cost minimization, we take the firm's cost (F)*1.0 and add to it λ times the constraint of producing q units. But in this case we use " J " for the Lagrange multiplier. We find the Lagrangean to be

$w\,L + v\,K + J\left(Q - K^a\,L^b\right)$.

We find the first order conditions by differentiating the Lagrangean with respect to K, L, and J, respectively, to find:

$$v - \frac{J\,K^a\,a\,L^b}{K} = 0$$

$$w - \frac{J\,K^a\,L^b\,b}{L} = 0$$

$$Q - K^a\,L^b = 0$$

These equations in many instances can be solved by taking the first two equations and moving the Lagrange multiplier term to the right hand side, and then dividing one equation by the other. The main purpose of this step is the cancel the Lagrange multiplier on the right hand side of the equation, as follows:

$$v\,/\,w = \left(\;-\frac{K^a\,a\,L^b}{K}\;\cdot J\right) / \left(\;-\frac{K^a\,L^b\,b}{L}\;\cdot J\right)$$

or $\frac{v}{w} = \frac{a\,L}{K\,b}$. We take this equation along with the last equation to find our solutions $K = \left(\frac{a\,w}{b\,v}\right)^{\left(\frac{a}{a+b}\right)} Q^{\left(\frac{1}{a+b}\right)}$ and

$$L = \left(\frac{b\,v}{a\,w}\right)^{\left(\frac{b}{a+b}\right)} Q^{\left(\frac{1}{a+b}\right)}.$$

To check that this is a minimum, we need to find the bordered Hessian, evaluate it at the solution, and check that its determinant is negative. We find the bordered Hessian by differentiating the Lagrangean again with all the variables, and arranging these second derivatives in a matrix. The bordered Hessian is

$$\begin{bmatrix} -\frac{J\,K^a\,a^2\,L^b}{K^2} + \frac{J\,K^a\,a\,L^b}{K^2} & -\frac{J\,K^a\,a\,L^b\,b}{K\,L} & -\frac{K^a\,a\,L^b}{K} \\ -\frac{J\,K^a\,a\,L^b\,b}{K\,L} & -\frac{J\,K^a\,L^b\,b^2}{L^2} + \frac{J\,K^a\,L^b\,b}{L^2} & -\frac{K^a\,L^b\,b}{L} \\ -\frac{K^a\,a\,L^b}{K} & -\frac{K^a\,L^b\,b}{L} & 0 \end{bmatrix},$$

which is evaluated at the solution $(K, L) = \left(\left(\frac{a\,w}{b\,v}\right)^{\left(\frac{a}{a+b}\right)} Q^{\left(\frac{1}{a+b}\right)}, \left(\frac{b\,v}{a\,w}\right)^{\left(\frac{b}{a+b}\right)} Q^{\left(\frac{1}{a+b}\right)} \right)$; and finally, taking the determinant, we find

$$-J\left(\left(\frac{a\,w}{b\,v}\right)^{\left(\frac{a}{a+b}\right)} Q^{\left(\frac{1}{a+b}\right)}\right)^{(3\,a)} a \left(\left(\frac{b\,v}{a\,w}\right)^{\left(\frac{b}{a+b}\right)} Q^{\left(\frac{1}{a+b}\right)}\right)^{(3\,b)} b\,(a + b)\left(\frac{a\,w}{b\,v}\right)^{\left(-2\,\frac{a}{a+b}\right)} Q^{\left(-\frac{4}{a+b}\right)} \left(\frac{b\,v}{a\,w}\right)^{\left(-2\,\frac{b}{a+b}\right)}$$

which is indeed negative, giving us a minimum.