Supplementary Material
for
"Consistent Re-identification in a Camera Network"

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Overview

This supplementary material contains the following:

- Normalized Area Under Curves (nAUC) values of the CMC curves on the WARD and RAiD datasets
- Formulation of the optimization problems as standard Binary Integer Programs (BIP)
- Equivalence of the two optimization problems (*i.e.*, NCR with same and variable set of persons across cameras) is proved. NCR with same set of persons across cameras can be derived from the optimization problem addressing NCR with variable number of people if the condition that the same set of persons appear in all the cameras is imposed on the later problem

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1 Comparison of nAUC Values

Table 1: Comparison of NCR with state-of-the-art methods on the WARD dataset in terms of the nAUC values.

Camera pair	SDALF	WACN	ICT	FT	NCR on ICT	NCR on FT
1-2	0.6487	0.7328	0.8780	0.9136	0.8835	**0.9317**
1-3	0.6825	0.7496	0.8240	0.8905	0.8299	**0.8981**
2-3	0.7206	0.7966	0.8881	0.9278	0.8910	**0.9330**

For all 3 camera pairs, 'NCR on FT' gives the best nAUC values.

Table 2: Comparison of NCR with state-of-the-art methods on the RAiD dataset in terms of the nAUC values.

Camera pair	SDALF	WACN	ICT	FT	NCR on ICT	NCR on FT
1-2	0.7987	0.9072	0.9138	0.9220	**0.9373**	0.9345
1-3	0.6576	0.6979	0.8145	0.8110	**0.8660**	0.8618
1-4	0.7274	0.7674	0.8413	0.8523	0.8790	**0.8885**
2-3	0.7802	0.8057	0.8328	0.8648	0.8700	**0.9008**
2-4	0.7956	0.8441	0.8615	0.9010	0.9008	**0.9210**
3-4	0.8014	0.8256	0.8813	0.8943	0.8990	**0.9138**

For all 6 camera pairs, either 'NCR on ICT' or 'NCR on FT' give the best nAUC values. Also for the 2 cases where 'NCR on ICT' gives better nAUC values than 'NCR on FT', the difference is in the 3^{rd} place of decimal.

2 Standard BIP Formulation of the Optimization Problems

For a matrix $\mathbf{A} = [\mathbf{a}_1\, \mathbf{a}_2\, \cdots\, \mathbf{a}_n] \in \mathbb{R}^{a \times b}$, let us define the vectorization operator $vec\,(.)$ as, $vec\,(\mathbf{A}) = [\mathbf{a}_1^T\, \mathbf{a}_2^T\, \cdots\, \mathbf{a}_n^T]^T = \underline{\mathbf{A}}$ (say) $\in \mathbb{R}^{ab}$. Now, the global similarity score in eqn. (3) of the main paper can be written as,

$$\begin{aligned}\mathbf{C} &= \sum_{\substack{p,q=1\\p<q}}^{m} (vec\,(\mathbf{C}^{(p,q)}))^T vec\,(\mathbf{X}^{(p,q)}) \\ &= \sum_{\substack{p,q=1\\p<q}}^{m} (\underline{\mathbf{C}}^{(p,q)})^T \underline{\mathbf{X}}^{(p,q)} \end{aligned} \tag{1}$$

Clubbing all the vectorized $\mathbf{C}^{(p,q)}$s in a single vector we can write,

$$\underline{\mathbf{C}} = [(\underline{\mathbf{C}}^{(1,2)})^T (\underline{\mathbf{C}}^{(1,3)})^T \cdots (\underline{\mathbf{C}}^{(1,m)})^T (\underline{\mathbf{C}}^{(2,3)})^T (\underline{\mathbf{C}}^{(2,4)})^T \cdots (\underline{\mathbf{C}}^{(2,m)})^T \cdots\cdots (\underline{\mathbf{C}}^{(m-1,m)})^T]^T \tag{2}$$

Similarly, all the vectorized $\mathbf{X}^{(p,q)}$s can be clubbed as,

$$\underline{\mathbf{X}} = [(\underline{\mathbf{X}}^{(1,2)})^T (\underline{\mathbf{X}}^{(1,3)})^T \cdots (\underline{\mathbf{X}}^{(1,m)})^T (\underline{\mathbf{X}}^{(2,3)})^T (\underline{\mathbf{X}}^{(2,4)})^T \cdots (\underline{\mathbf{X}}^{(2,m)})^T \cdots\cdots (\underline{\mathbf{X}}^{(m-1,m)})^T]^T \tag{3}$$

Using eqn. (2) and (3), eqn. (1) can be written compactly as,

$$\mathbf{C} = \underline{\mathbf{C}}^T \underline{\mathbf{X}} \tag{4}$$

Let us express eqn. (2) of the main paper in terms of $\underline{\mathbf{X}}^{(p,q)}$.

$$\begin{aligned} &\sum_{j=1}^{n} x_{i,j}^{p,q} = x_{i,1}^{p,q} + x_{i,2}^{p,q} \cdots + x_{i,n}^{p,q} = 1\ \forall i = 1 \text{ to } n \\ \implies &[1, 1, \cdots, 1][x_{i,1}^{p,q}, x_{i,2}^{p,q}, \cdots, x_{i,n}^{p,q}]^T = 1\ \forall i = 1 \text{ to } n \end{aligned} \tag{5}$$

and

$$\begin{aligned} &\sum_{i=1}^{n} x_{i,j}^{p,q} = x_{1,j}^{p,q} + x_{2,j}^{p,q} \cdots + x_{n,j}^{p,q} = 1\ \forall j = 1 \text{ to } n \\ \implies &[1, 1, \cdots, 1][x_{1,j}^{p,q}, x_{2,j}^{p,q}, \cdots, x_{n,j}^{p,q}]^T = 1\ \forall j = 1 \text{ to } n \end{aligned} \tag{6}$$

Writing eqns. (5) and (6) respectively for all rows and columns of $\mathbf{X}^{(p,q)}$ we get,

$$\left[\begin{array}{cccccccccccccccc} \multicolumn{5}{c}{\overbrace{\hphantom{1\,0\,0\cdots0}}^{n \text{ columns}}} & \multicolumn{5}{c}{\overbrace{\hphantom{1\,0\,0\cdots0}}^{n \text{ columns}}} & & \multicolumn{5}{c}{\overbrace{\hphantom{1\,0\,0\cdots0}}^{n \text{ columns}}} \\ 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1 & \cdots & 0 & 0 & 0 & \cdots & 1 \\ \hline 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 1 & 1 & \cdots & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & 1 & 1 & \cdots & 1 \end{array}\right] \begin{bmatrix} x^{p,q}_{1,1} \\ x^{p,q}_{2,1} \\ \vdots \\ x^{p,q}_{n,1} \\ x^{p,q}_{1,2} \\ x^{p,q}_{2,2} \\ \vdots \\ x^{p,q}_{n,2} \\ \vdots \\ x^{p,q}_{1,n} \\ x^{p,q}_{2,n} \\ \vdots \\ x^{p,q}_{n,n} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} \tag{7}$$

Writing the matrix as $\mathbf{A}$ and the vector as $\underline{\mathbf{X}}^{(p,q)}$ in eqn. (7) the equation can be written as,

$$\mathbf{A}\underline{\mathbf{X}}^{(p,q)} = \underline{\mathbf{1}} \tag{8}$$

where, $\underline{\mathbf{1}}$ is a vector consisting of all 1's.

Relation (8) can be written for all the camera pairs, *i.e.*, $\forall p, q = [1, \cdots, m], p < q$. In a block matrix form these can be written as,

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \cdots & \mathbf{A} \end{bmatrix} \begin{bmatrix} \underline{\mathbf{X}}^{(1,2)} \\ \underline{\mathbf{X}}^{(1,3)} \\ \vdots \\ \underline{\mathbf{X}}^{(1,m)} \\ \underline{\mathbf{X}}^{(2,3)} \\ \underline{\mathbf{X}}^{(2,4)} \\ \vdots \\ \underline{\mathbf{X}}^{(2,m)} \\ \vdots \\ \underline{\mathbf{X}}^{(m-1,m)} \end{bmatrix} = \underline{\mathbf{1}} \tag{9}$$

Denoting the matrix as $\underline{\mathbf{A}}$ and the vector as $\underline{\mathbf{X}}$, eqn. (9) can be written as,

$$\underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}} \tag{10}$$

Let us express eqn. (7) of the main paper in a similar way. The equation can be written as,

$$-x^{p,q}_{i,j} + x^{p,r}_{i,k} + x^{r,q}_{k,j} \leq 1 \tag{11}$$

Let us, first, write the set of these constraints for a particular triplet of cameras denoted by p, q, r with $p < r < q$. For this let us introduce some more notations. Let the j^{th} column of $\mathbf{X}^{(p,q)}$ be denoted as,

$$\mathbf{x}_j^{(p,q)} = [x_{1,j}^{p,q}, x_{2,j}^{p,q}, \cdots, x_{n,j}^{p,q}]^T$$

Let $\underline{\mathbf{1}}_i$ denote a vector of all 0's except a 1 at the i^{th} position. Let $\underline{\mathbf{0}}$ denote a vector of all 0's.

Keeping $i = 1; j = 1$ we, first, vary $k = 1, 2, \cdots, n$ in eqn. (11) to get,

$$\begin{aligned}
-x_{1,1}^{p,q} + x_{1,1}^{p,r} + x_{1,1}^{r,q} &\leq 1 \quad \text{for } k = 1 \\
-x_{1,1}^{p,q} + x_{1,2}^{p,r} + x_{2,1}^{r,q} &\leq 1 \quad \text{for } k = 2 \\
-x_{1,1}^{p,q} + x_{1,3}^{p,r} + x_{3,1}^{r,q} &\leq 1 \quad \text{for } k = 3 \\
&\vdots \\
-x_{1,1}^{p,q} + x_{1,n}^{p,r} + x_{n,1}^{r,q} &\leq 1 \quad \text{for } k = n
\end{aligned}$$

The above set of of equations can also be written as,

$$\left[\begin{array}{cccc|ccccc|cccc}
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} \\
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_2 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} \\
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_3 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} \\
\vdots & \vdots & \vdots \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \ddots & \vdots \\
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{1}}_1 & \underline{\mathbf{1}}_n & \underline{\mathbf{0}} & \underline{\mathbf{0}} \cdots & \underline{\mathbf{0}}
\end{array}\right]
\left[\begin{array}{c}
\mathbf{x}_1^{(p,q)} \\ \mathbf{x}_2^{(p,q)} \\ \mathbf{x}_3^{(p,q)} \\ \vdots \\ \mathbf{x}_n^{(p,q)} \\ \hline
\mathbf{x}_1^{(p,r)} \\ \mathbf{x}_2^{(p,r)} \\ \mathbf{x}_3^{(p,r)} \\ \vdots \\ \mathbf{x}_n^{(p,r)} \\ \hline
\mathbf{x}_1^{(r,q)} \\ \mathbf{x}_2^{(r,q)} \\ \mathbf{x}_3^{(r,q)} \\ \vdots \\ \mathbf{x}_n^{(r,q)}
\end{array}\right] \leq \underline{\mathbf{1}} \qquad (12)$$

For the same camera triplet (p, q, r), appending the rows corresponding to $i = 1, j = 2$ and $k = 1, 2, \cdots n$ we get,

$$
\left[\begin{array}{ccccc|ccccc|ccccc}
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_2 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_3 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
-\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{1}}_1 & \underline{\mathbf{1}}_n & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
\hline
\underline{\mathbf{0}} & -\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
\underline{\mathbf{0}} & -\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_2 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
\underline{\mathbf{0}} & -\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_1 & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{1}}_3 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\underline{\mathbf{0}} & -\underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{1}}_1 & \underline{\mathbf{0}} & \underline{\mathbf{1}}_n & \underline{\mathbf{0}} & \cdots & \underline{\mathbf{0}}
\end{array}\right]
\left[\begin{array}{c}
\mathbf{x}_1^{(p,q)} \\ \mathbf{x}_2^{(p,q)} \\ \mathbf{x}_3^{(p,q)} \\ \vdots \\ \mathbf{x}_n^{(p,q)} \\ \hline
\mathbf{x}_1^{(p,r)} \\ \mathbf{x}_2^{(p,r)} \\ \mathbf{x}_3^{(p,r)} \\ \vdots \\ \mathbf{x}_n^{(p,r)} \\ \hline
\mathbf{x}_1^{(r,q)} \\ \mathbf{x}_2^{(r,q)} \\ \mathbf{x}_3^{(r,q)} \\ \vdots \\ \mathbf{x}_n^{(r,q)}
\end{array}\right]
\leq \underline{\mathbf{1}} \tag{13}
$$

Progressing in this way for all triplets of cameras and all persons, and denoting the resulting matrix (similar to the one in the left hand side of eqn. (13)) as $\underline{\mathbf{B}}$, we get the loop constraints as,

$$
\underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}} \tag{14}
$$

where, the number of rows of $\underline{\mathbf{B}}$ is the total number of loop constraint equations which is $\binom{m}{3}\binom{n}{2}(n-2) = \frac{m(m-1)(m-2)n(n-1)(n-2)}{12}$

Thus, using the objective function from eqn. (4) and the constraints from eqns (10) and (14) we can write the binary integer program in standard form as,

$$
\begin{aligned}
&\underset{\underline{\mathbf{X}}}{\operatorname{argmax}}\ \underline{\mathbf{C}}^T\underline{\mathbf{X}} \\
&\text{subject to } \underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}} \\
&\underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}} \\
&\underline{\mathbf{X}} \text{ is composed of binary variables.}
\end{aligned} \tag{15}
$$

where, the dimensions of the matrices and vectors are as follows,

$\underline{\mathbf{C}}$ is $\frac{m(m-1)n^2}{2} \times 1$, $\underline{\mathbf{A}}$ is $m(m-1)n \times \frac{m(m-1)n^2}{2}$ and $\underline{\mathbf{B}}$ is $\frac{m(m-1)(m-2)n(n-1)(n-2)}{12} \times \frac{m(m-1)n^2}{2}$.

In a similar way, the standard binary integer program formulation of the optimization problem given in eqn. (11) of the main paper for a variable number of targets across cameras can be expressed as,

$$\begin{aligned} \underset{\underline{\mathbf{X}}}{\operatorname{argmax}}\,(\underline{\mathbf{C}}^T - k\underline{\mathbf{1}}^T)\underline{\mathbf{X}} \\ \text{subject to } \underline{\mathbf{A}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}} \\ \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}} \\ \underline{\mathbf{X}} \text{ is composed of binary variables.} \end{aligned} \tag{16}$$

where $\underline{\mathbf{C}}, \underline{\mathbf{X}}, \underline{\mathbf{A}}$ and $\underline{\mathbf{B}}$ are as explained above with the dimensions of them changed to incorporate variable number of persons across cameras.

Equivalence of the two Formulations in Sections 3.3 and 3.4 of the Main Paper

The overall optimization problem (ref eqn. (8) of the main paper) as given in section 3.3 of the main paper and the same (ref eqn. (11) of the main paper) as given in section 3.4 are formulated as standard binary programs in the previous section as eqn. (15) and (16) respectively. Now we will show that the problem expressed by eqn. (16) is equivalent to the problem expressed by eqn. (15) under the condition that the same set of persons appear in all the cameras. Let $\underline{\mathbf{X}}^*$ be the optimal solution to the problem expressed by eqn. (16). To prove the equivalence, we have to show that $\underline{\mathbf{X}}^*$ also maximizes the problem expressed by eqn. (15).

Since $\underline{\mathbf{X}}^*$ maximizes the objective function under the constraints as expressed by eqn. (16), we can write,

$$\begin{aligned} (\underline{\mathbf{C}}^T - k\underline{\mathbf{1}}^T)\underline{\mathbf{X}}^* \geq (\underline{\mathbf{C}}^T - k\underline{\mathbf{1}}^T)\underline{\mathbf{X}} \\ \text{for } \{\underline{\mathbf{X}} : \underline{\mathbf{A}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}, \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}\} \\ \text{where both } \underline{\mathbf{X}}^* \text{ and } \underline{\mathbf{X}} \text{ are composed of binary variables.} \end{aligned} \tag{17}$$

Since $\{\underline{\mathbf{X}} : \underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}}, \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}\} \subset \{\underline{\mathbf{X}} : \underline{\mathbf{A}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}, \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}\}$, the relation (17) holds true for the feasible set of eqn. (15), *i.e.*,

$$\begin{aligned} (\underline{\mathbf{C}}^T - k\underline{\mathbf{1}}^T)\underline{\mathbf{X}}^* \geq (\underline{\mathbf{C}}^T - k\underline{\mathbf{1}}^T)\underline{\mathbf{X}} \\ \text{for } \{\underline{\mathbf{X}} : \underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}}, \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}\} \\ \text{with both } \underline{\mathbf{X}}^* \text{ and } \underline{\mathbf{X}} \text{ composed of binary variables.} \\ \implies \underline{\mathbf{C}}^T\underline{\mathbf{X}}^* - k\underline{\mathbf{1}}^T\underline{\mathbf{X}}^* \geq \underline{\mathbf{C}}^T\underline{\mathbf{X}} - k\underline{\mathbf{1}}^T\underline{\mathbf{X}} \\ \text{for } \{\underline{\mathbf{X}} : \underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}}, \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}\} \\ \text{with both } \underline{\mathbf{X}}^* \text{ and } \underline{\mathbf{X}} \text{ composed of binary variables.} \end{aligned} \tag{18}$$

Now for all $\underline{\mathbf{X}}$ and $\underline{\mathbf{X}}^*$ that satisfy $\underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}}$ (*i.e.*, for the case when the same set of n persons appear in all m cameras),

$$\underline{\mathbf{1}}^T\underline{\mathbf{X}}^* = \underline{\mathbf{1}}^T\underline{\mathbf{X}} = \text{Number of camera pairs } \times \text{ Number of persons} = \binom{m}{2}n$$

This is because, each row and column of the assignment matrix for each camera pair contains exactly one 1, resulting in the sum of all elements of the assignment matrices being n.

Using the above relation in eqn. (18) we get,

$$\begin{aligned} \underline{\mathbf{C}}^T\underline{\mathbf{X}}^* \geq \underline{\mathbf{C}}^T\underline{\mathbf{X}} \\ \text{for } \{\underline{\mathbf{X}} : \underline{\mathbf{A}}\underline{\mathbf{X}} = \underline{\mathbf{1}}, \underline{\mathbf{B}}\underline{\mathbf{X}} \leq \underline{\mathbf{1}}\} \\ \text{with both } \underline{\mathbf{X}}^* \text{ and } \underline{\mathbf{X}} \text{ composed of binary variables.} \end{aligned}$$

Therefore, $\underline{\mathbf{X}}^*$ also maximizes the problem (15), thus proving the equivalence.