**A daily model for infections such as Covid-19, Version 2.2**

**1 The model**

1.1 In this note I describe a daily model for infections such as Covid-19 which I have constructed in Excel and make available for anyone else to use. It is based on my actuarial experience with income protection insurance (which provides a daily benefit while the insured is sick) and with modelling for AIDS in the 1980s. The model is a simple one, in that it assumes one homogeneous population, with no separation by age or sex or any other factor, but it allows me to get it onto one Excel worksheet.

1.2 The very simple continuous model which produces a logistic curve: eat / (1 + eat) for the total number of infections assume implicitly that an infected person is always infectious, but with many diseases an infected person may be infectious for only a limited period, and generally ceases to be infectious on death. This model allows for these features.

1.3 My model is a state-space model, with daily possible transitions between states. Time, *t*, is measured in days. The period of infection, *d*, is also measured in days. The states are *H*, healthy, *I*, infected and *D*, dead. The whole population starts initially in state *H*, and on some chosen first day, *t* = 1, a new infection appears (from where, I do not specify). By infections from those who are infected and infectious, people can transfer to state *I* on becoming infected. They may then die, and go to state *D*. I ignore entirely other deaths in the population.

1.4 My model is probabilistic, but does not allow for random variation. I calculate expected values throughout, and make no assumptions about the variation about those values. This does not matter when the numbers become large, but it may appear incongruous when the numbers are small and one speaks of fractional numbers of deaths or inceptions; every time I mean an expected number.

1.5 Those infected are kept separate according to the number of days infected, starting in day *d* = 1, and going up to *d* = 21, and all those infected for more than 21 days are put in a separate bundle. The number 21 is arbitrary, but it seems enough for Covid-19. At the end of each day an infected person may die. Those infected may be infectious or not infectious on any day; normally one would assume that they were infectious for a consecutive number of days at the beginning of the illness, and this number can be varied. But there could be days at the start of infection when they are not infectious. The dead are assumed to be not infectious.

1.6 The number of new infections each day depends on the number of infected persons on that day, *t*, and duration, *d*, times a factor that depends on the day, *t*, (so can be varied with time), times the proportion of non-infected in the total population, times an indicator to show whether they are still infectious or not.

1.7 To explain the detail I now refer to the Excel workbook Cv19DailyModel\_ V2.2.xlsx, worksheet Calculations. Below the headings there is one row for each day, *t*. Days, starting at day 0, are numbered in column D, and this is repeated for convenience in columns AL and BR. Move ahead to column I, where the number of healthy persons each day, *H*(*t*), is given. The number in the total population at the start, *H*(0), is placed in cell I11. I use bold for all headings and bold with a pink background for all input variables, so that it is easy to identify them.

1.8 The number infected and still alive on day *t* at duration *d* is denoted *I*(*t*, *d*), and is given in Columns N (for *d* = 1) to AI (for *d* ≥ 22). For *t* = 0 they are all zero. The total *I*(*t*) is given in column AJ. On day 1 a number of initial cases, usually 1, is put into *I*(1,1) in cell I12; or one can start at a later day if desired. If one were to insert zero there, there would be no infections at all.

1.9 Apart from the first duration day (*d* = 1) which I deal with later, the number infected at time *t*, duration *d*, moves to time *t*+1, day *d*+1, but may die or survive (but still infected) on the way. Deaths are assumed to take place at the end of each day, and infections at the beginning of each day.

1.10 In row 6, columns N to AI, are the values of *m*(*d*), the probability that a person who is infected at duration *d* dies at the end of that day, for *d* = 1 to *d* = 21. Once infected persons get into the “over 21” box, they are assumed not to die (from this disease). The probability of survival from day *t* to day *t*+1 is therefore 1 – *m*(*d*). Thus we get the expected number of infected at *t*+1, *d*+1, *I*(*t*+1, *d*+1) = *I*(*t*, *d*).(1 – *m*(*d*)). So in cell O13, for *t* = 2, *d* = 2, we get the survivors of *t* = 1, *d* = 1. With *I*(1, 1) = 1 in cell N12 and *m*(1) = 0.04 in cell N6, we get 0.96 survivors (expected). Look along to cell AO13 and we get the number of new deaths that day, *D*(*t*, *d*) = 0.04 (expected), calculated as *D*(*t*+1, *d*+1) = *I*(*t*, *d*).*m*(*d*). I shall return to the deaths later.

1.11 When we reach column AI for the infected over 21 days, the calculation needs to include those carried forward from the previous day in that box, as well as those ones entering that box from day 21. So *I*(*t*+1, *d*>21) = *I*(*t*, *d*>21) + *I*(*t*, 21).(1 – *m*(21)). The first non-zero cell in that is AI33, which includes zero from AI32 plus the 0.96 carried forward (with no further deaths with the given parameters) from the original infected in cell AH32 times the survival probability rate (1 – *m*(21)), which in the example is 1. Column AJ gives the total of all those infected at different durations, *I*(*t*), omitting the deaths.

1.12 Going further right in the sheet we have the numbers of new deaths at each duration, *D*(*t*, *d*). If the value of *m*(*d*–1) is zero, then there are no new deaths for that day. There are no new deaths for day *d* = 1, so the cells in column AI are blank. Depending on the mortality rates assumed, there may be deaths for each day up to *d* = 21, in columns AO to BH. There may be deaths at the end of day 21, and they are placed in column BI. Then the number of deaths on that day is summed in column BJ and the cumulative number of deaths so far, denoted *DT*(*t*), is in Column BK.

1.13 In column BL is the total number of infected plus the total number of dead so far, *I*(*t*) + *DT*(*t*), which is the same number as those in column K, which I come to below. Tse numbers can be compared with the published number of cases so far, just as the total deaths so far can be compared. In column BN is the “Ratio per week”, the same as in column L After day *t* = 7 this it is the number of infections so far divided by the number seven days previously, which is a useful indicator of the initial rate of growth in the population. This can be compared with the figures form real populations which are published. In Column BO is shown the total deaths so far divided by the total cases so far, as a percentage. which can also be compared with the actual experience.

1.14 In column BQ is a “check total”. This is the total number of Healthy, Infected, and Dead so far by day *t,* together with the sum of the adjustments, *X*(*t*), described below, and equals *H*(*t*) + *I*(*t*) + *DT*(*t*) + Sum(*X*(*t*) up to time *t*),. This should always equal the starting number, *H*(0), and is a useful check on the arithmetic.

1.15 I now come to the calculation of new infections. In column BT is shown the proportion not infected, *p*(*t*), which equals the number of healthy on that day *H*(*t*) divided by the number living on that day, *H*(*t*) + *I*(*t*), so excluding the deaths. In columns BU to CP are shown the numbers of new infections arising from those infected on each day for *d* = 1 to *d* > 21, denoted *N*(*t*, *d*). This depends on the number infected at duration *d*, *I*(*t*, *d*), the infection rate, *r*(*t*), the probability that that a person of duration *d* can be infectious, *j*(*d*), and the proportion of the population available to be infected, *p*(*t*).

1.16 The infection rate, *r*(*t*), can vary with the time, *t*, but in this example it is constant. It is shown in column E, and the first value in cell E12 is copied down. But to change it, one can insert a number in the cell for the appropriate day and it will be copied thereafter. For convenience I shown in column F what I call theoretical R7. = *exp*(7.*r*(*t*)), the weekly multiplier in a continuous model, This can be compared with the weekly rate in the actual model in Column L. In column G I give “R0” which I come to below.

1.17 The values of the probabilities that a person of duration *d* can be infectious, *j*(*d*), are shown in row 5, Columns N to AI. They are shown there for convenience in altering them, but are not used until later, when they are copied into row 7 columns BU to CP. I have used values only of 1 and 0 to show infectious or not infections, but intermediate values could be used to show a person becoming less or more infectious as the duration of infection goes by.

1.18 We then have the formula: *N*(*t*, *d*) = *I*(*t*, *d*).*r*(*t.*.*j*(*d*).*p*(*t*). The values are shown in columns BU to CP and summed in column CQ. This shows *N*(*t*), the number of new infections that day. I assume that they start the infection on the following day, so *I*(*t*+1, 1) = *N*(*t*). the value in column CQ is copied back into column N in the row below, and we start the next day. At this point the number of Healthy is updated, by putting *H*(*t*) = *H*(*t*–1) – *N*(*t*–1)

1.19 I now describe two features additional to the original model. It seems useful to be able to adjust the number of Healthy to reflect the domestic isolation of many people with recent measures. In column J one can put a number, X(t), to represent a deduction from H(t), the current number of healthy. Thus we put: *H*(*t*) = *H*(*t*–1) – *N*(*t*–1) – *X*(*t*). The Check sum also needs to take these into account.

1.20 The other new feature is to calculate the factor R0, which epidemiologists use. This is described as the number of new persons that one infected person would infect, assuming there were none already infected. This requires some calculation. First, I use the mortality rates *m*(*d*) to calculate for each duration *d* the proportion of survivors to that day. I call this proportion *p*(*d*) and the values are shown in row 7, column s N to AJ. In cell N7 is the value of p(1), put as 1,0, . Subsequent values are calculated in succeeding cells by the formula: *p*(*d*+1) = *p*(*d*).(1 – *m*(*d*)). Next I calculate in row 8 a series of numbers for *q*(*d*) = *p*(*d*).*j*(*d*)., the intensity of infection on day *d*. I sum these numbers to get *Q*, in cell N9, which in the example is 9.64./ This number is then multiplied by *r*(*t*) for day *t*, and put in column G. In the example it is 2.e1 which is the same area as I have seen stated for R0.

1.21 The input variables, or parameters, that one can readily change are:

(a) The initial susceptible population *H*(0) in cell I11

(b) The initial number of inceptions on day 1, in cell I12.

(c) The values of *j*(*d*), the infectiousness at duration *d*, in cells I7 to AD7

(d) The values of *m*(*d*), the probabilities of death at duration *d*, in cells I8 to AC8.

(e) The values of *r*(*t*), the infection rate at time *t*, in cells E12 and below; nut this can be readily changed for later values of *t*.

(f) The value of the calendar date for time *t* = 0, in cell B11, see below.

(g) Any desired adjustments to the healthy, *X*(*t*), in Volume J.

The values in all other cells are calculated from these inputs.

1.22 In addition, in column B is a series of calendar dates, staring with some input date at *t* = 0. This needs to be chosen after all other input values have been inserted, and it depends on the results. In columns K and BM one sees the total number infected (including dead) so far. One can compare these with the latest published numbers for any given country, and set the initial date in cell B11 so that the numbers may match up at the current date. In addition in column C are given “weeks”, in the sense of the given day divided by 7, so that one can see for example how many weeks ahead the peak might be, or the numbers of new infections drop to zero or nearly zero.

1.21 There is also a Chart of New Infections daily, but one can add whatever charts one likes.

**2 The numbers|**

2.1 I need to put in specimen numbers, and these can be changed as you wish. However, I explain here why I have chosen the ones that are there in this version. You may disagree with my choice, but the point of this model is that you can easily try out whatever you would like to put in. It is interesting to try out quite different numbers, for example with very high death rates, to check that such a disease would, with this sort of model, die out very quickly. In fact the initial value are as described in Section 1, but you need to change them as described below to get to the final results.

2.2 I have found a useful and comprehensive data source published by the European Centre for Disease Control at:

<https://www.ecdc.europa.eu/en/publications-data/download-todays-data-geographic-distribution-covid-19-cases-worldwide>

This gives daily numbers for a large number of countries in a basic, but useable, format. I have transposed this into a different, more informative, format and the corresponding Excel file, which includes macros, is available.

2.3 I have chosen numbers for this model that reflect the position in the United Kingdom, according to this data, as of 21 March 2020. The position day by day changes so rapidly that these will be out of data before I finish writing, but I shall persevere. At that date there had been 3,983 reported cases and 177 reported deaths. The deaths were 4.44% of the number of cases. The number of cases since seven days previously (14 March) had multiplied by 4.63 and the number of deaths by 17.7.

2.4 The first input is (a) the initial susceptible population *H*(0) in cell I11. I have put 5,000,000. Why not put in the whole population of the UK? The first reason is that there seems to be evidence that a large number of people can get the infection, and suffer from it very mildly, or not even notice it. It is speculated that there might be 10 times as many of these as get COVID-19 sufficiently badly for it to be noticed. But anyone acutely ill, needing hospitalisation, is almost sure to be recorded, and deaths will also be recorded. So I treat the very mildly affected as if they were “immune”, and they are left out of the calculations.

2.5 Experiments show that if we put in 10 times the value of *H*(0) and 10 persons on the starting date instead of 1, and then reduce the mortality rate to one tenth of its initial value, and further assume that 90% of cases are so mild as not to be reported, but that all the deaths are serious enough to be reported , then the results are fairly similar to what we get with the original assumptions. The number of reported infected (still alive) is a bit more, the number of deaths is much the same, and the total number of deaths is identical, with one tenth the rate times 10 times the population.

2.6 Secondly, there are some areas and institutions that could keep entirely clear of the virus, remote islands, remoter country areas, monasteries, prisons, secure mental hospitals. All these could make themselves immune with sufficiently little outside contact, and careful checking, though if they were to be infected the disease might spread rapidly. So I put in 5,000,000 as *H*(0), less than 10% of the UK population.

2.7 The next input is (b), the initial number of inceptions on day 1, in cell I12. I put this at 1, but any other integer would do. Apart from the first person worldwide, for all countries the first infected person is an import. In fact there seem to have been several independent imports into the UK, but in order to investigate the pattern in one country, putting 1 is good enough.

2.8 The next input is (c) the values of *j*(*d*), the infectiousness at duration *d*, in cells N5 to AI5. I have put these at 1 for each day from 1 to 10. I do not know what the best medical opinion is on this, but if quarantine for 14 days is considered sufficient, then the period of infectiousness should be less than this. It is easy to assume 7 or 14 or any other number of days, or to assume that there are different degrees of infectiousness by putting fractional numbers for *j*(*d*), or that there is a delay before an infected person is infectious, by putting zero for the first or subsequent days. However the values of *j*(*d*) remain the same for all days *t*. We get variation by time by varying *r*(*t*), as discussed below.

2.9 The next input is (d) the values of *m*(*d*), the probabilities of death at duration *d*, in cells N6 to AH6. The deaths so far recorded in the U.K. have been about 4% of recorded cases, and this number applies also in China and world-wide, though in Italy it has been much higher. I have seen it argued that the real rate is much lower, because of the unrecorded mild cases; but I feel that we can only look at recorded cases,, and not speculate on the unknown number of unrecorded ones It is also possible that the death rate will turn out to be a little higher, because people do not all die immediately on getting the disease, so there may be deaths in the pipeline from cases already reported.

2.10 I initially use a total mortality rate of 0.04, which neds to be spread over the values of *m*(*d*) in some way. I do not know anything about the incidence of death by duration of illness. At one extreme one could put *m*(1) = 0.04 and all other values zero, or one could put it at the end of the period of infectiousness, or spread it over the day, putting say 0.004 for *m*(1) to *m*(10) inclusive. I decide finally after further investigation of the actual progress in the UK, as described below. But to start with I put *m*(1) = 0.04 in cell N6 and all other values of *m*(*d*) to zero.

2.11 The next, and very important input is (e) the values of *r*(*t*), the infection rate at time *t*, in cells E12 and below. To set this I need also to consider input (f) the value of the calendar date for time *t* = 0, in cell B11. The first recorded cases in the UK were two on 31 January, so I put *t* = 0 on 30 January, so 21 March is day 51 and I try to match results with that.

2.12 I initially try a value for *r*(*t*), say 0.2, for all values of *t*. I then look at the number of cases so far, the infected plus dead, in column BG. The actual number of cases in the UK reported as at 21 March was 3,983, with 177 deaths. My results for 21 March are in row 62. I get the total number of cases according to the model in cell BG62, and with *r*(*t*) = 0.2 it is it is 1,402, so it is much too low. By trial and error I get 3,812 cases with *r*(*t*) = 0 .22. You can see this by changing cell E12 to 0.220.

2.13 The total number of deaths so far, according to the model, is shown in column BF. The number of deaths as a proportion of the total number of cases is shown in column BI. With *m*(1) = 0.04, the percentage in column BI is always less than this to start with, because of the time lag even if deaths occur at the beginning of duration day 2., though eventually the percentage of deaths catches up. With *r*(*t*) = 0.222 and m(1) = 0.04 I get 130 deaths by 21 march. To match the 177 observed deaths by 21 March I need to increase the value of *m*(1). Changing the value of *m*(1) changes all the other numbers. After several trials and errors I use values *r*(*t*) = 0.218 and *m(*1) = 0.05 and get 4,020 cases and 171 deaths by 21 March, and you can replicate this by putting E12 to 0.222 and N6 to 0.05.

2.14 However, I now have much too low a rate of increase, R7, both for Cases and for Deaths. To get the observed high rate of increase I need to start later. I put a single 1 in cell N27 on 15 February, change *r*(*t*) to 0.3, *m*(1) to 0.06, and I get plausibly the right sort of numbers. See “CV19DailyModel Notes on |ADW01.docx” for more details.

2.15 With these values the number of new cases reaches a maximum of 303,954 on 21 April, and the maximum number of deaths is a day later, at 18,237 on 22 April. I would not be surprised if, with these numbers, the recording system were to collapse. Also the funeral systems could not cope, since the normal number of deaths per day in the UK is less than 2,000, with some seasonal variation. However, by 11 June the number of new cases expected by the model drops to less than 1.00, and the total number of deaths is 284,496. However, not every one of the initial *H*(0) has been infected, with 258,394 escaping infection altogether. This is because the infection period is limited to 10 days, and as the numbers go down, fewer and fewer infected are in this period.

2.16 As at the time of writing the various governments in the UK (including those of the devolved nations) have taken various steps to try to control the infection. I cannot estimate the effect of any of the measures. But they have to affect the epidemic either by reducing *r*(*t*) or by changing *H*(*t*). One can reduce the value of *r*(*t*) by putting a chosen value in column E opposite the relevant day. If, for example, you wished to set *r*(*t*) = 0.15 for 24 March onwards, put 0.15 in cell E65. It is copied on down for each cell thereafter. With this, one can see that the peak numbers of infections and deaths are very much down, to less than 50,000 and less than 3,000 in early July. But the disease continues to November. The number of deaths is down at about 165,000 and over 2.2 million remain healthy.

2.17 Changing *H*(*t*) can be done in this model. Assume that the isolation measures proposed put healthy people (and perhaps especially older ones, though I am not allowing for age) into the “immune” category. Imagine that this works with about half the population as 24 March.

Put *X*(54) as 2,500,000 in cell J65 The formula in cell G65 is “= I64 – N65 – J65”, that is the previous days healthy minus the day’s new infections minus the adjustment.. .If we include both adjustments, to *r*(*t*) and *H*(*t*), we get the peak a bit sooner, about 28 May, with the number of cases and deaths again reduced to about 24,000 and 1,500 per day. The epidemic lasts till early November, but the total number of death is almost halved to a little over 82,000. This is still a lot of deaths.

2.18 I must emphasize that I am not making specific forecasts. I am providing a tool that allows others to try out the results of hypothetical changes in what seem to me to be a plausible set of initial assumptions. If I made it much more realistic by stratifying the population by age, the model would need a full programme, which would be much harder for people to play with. Those who are happy changing the Excel may modify it as they wish. Others may just try altering certain values. My only request is that you recognise the authorship, and do not try copywriting it yourself to prevent others using it.

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