Information Consensus for Distributed Multi-Target Tracking
Supplementary Materials

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1. JPDAF: Information Form

The JPDAF estimation and covariance update equation are given in the following (Eqns. (15-16) in the main paper) which are then converted to the equivalent information form (Eqns. (18-19) in the main paper) below. The time index t has been dropped for simplicity.

$$\begin{aligned}
\mathbf{K}^j &= \mathbf{P}^{j-}{\mathbf{H}^j}^T{\mathbf{S}^j}^{-1} \\
&= \mathbf{P}^{j-}{\mathbf{H}^j}^T\left(\mathbf{H}^j\mathbf{P}^{j-}\mathbf{H}^{jT}+\mathbf{R}^j\right)^{-1} \\
&= \mathbf{P}^{j-}{\mathbf{H}^j}^T\left({\mathbf{R}^j}^{-1}-{\mathbf{R}^j}^{-1}\mathbf{H}^j\left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)^{-1}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\right) \quad \text{(using Matrix Inversion Lemma)} \\
&= \left(\mathbf{P}^{j-}-\mathbf{P}^{j-}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)^{-1}\right){\mathbf{H}^j}^T{\mathbf{R}^j}^{-1} \\
&= \left(\mathbf{P}^{j-}\left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)-\mathbf{P}^{j-}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)\left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)^{-1}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1} \\
&= \left(\mathbf{I}_p+\mathbf{P}^{j-}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j-\mathbf{P}^{j-}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)\left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)^{-1}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1} \\
&= \left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)^{-1}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1} \qquad (1)
\end{aligned}$$

$$\tilde{\mathbf{y}}^j = \mathbf{y}^j-(1-\beta^{j0})\mathbf{H}^j\hat{\mathbf{x}}^{j-} \qquad (2)$$

$$\begin{aligned}
\hat{\mathbf{x}}^{j+} &= \hat{\mathbf{x}}^{j-}+\mathbf{K}^j\tilde{\mathbf{y}}^j \\
&= \hat{\mathbf{x}}^{j-}+\left((\mathbf{P}^{j-})^{-1}+{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\mathbf{H}^j\right)^{-1}{\mathbf{H}^j}^T{\mathbf{R}^j}^{-1}\left(\mathbf{y}^j-(1-\beta^{j0})\mathbf{H}^j\hat{\mathbf{x}}^{j-}\right) \\
&= \hat{\mathbf{x}}^{j-}+\left(\mathbf{J}^{j-}+\mathbf{U}^j\right)^{-1}\left(\mathbf{u}^j-(1-\beta^{j0})\mathbf{U}^j\hat{\mathbf{x}}^{j-}\right) \\
&= \left(\mathbf{J}^{j-}+\mathbf{U}^j\right)^{-1}\left(\mathbf{J}^{j-}\hat{\mathbf{x}}^{j-}+\mathbf{U}^j\hat{\mathbf{x}}^{j-}+\mathbf{u}^j-(1-\beta^{j0})\mathbf{U}^j\hat{\mathbf{x}}^{j-}\right) \\
&= \left(\mathbf{J}^{j-}+\mathbf{U}^j\right)^{-1}\left(\mathbf{J}^{j-}\hat{\mathbf{x}}^{j-}+\mathbf{u}^j+\beta^{j0}\mathbf{U}^j\hat{\mathbf{x}}^{j-}\right) \qquad (3)
\end{aligned}$$

$$\begin{aligned}
\mathbf{P}^{j+} &= \mathbf{P}^{j-}-(1-\beta^{j0})\mathbf{K}^j\mathbf{S}^j{\mathbf{K}^j}^T+\mathbf{K}^j\tilde{\mathbf{P}}^j{\mathbf{K}^j}^T \\
&= \mathbf{P}^{j-}-\mathbf{K}^j\left((1-\beta^{j0})\mathbf{S}^j-\tilde{\mathbf{P}}^j\right){\mathbf{K}^j}^T \qquad (4)
\end{aligned}$$

where,

$$\tilde{\mathbf{P}}^j = \left(\sum_{n=1}^{l}\beta^{jn}\tilde{\mathbf{z}}^{jn}\left(\tilde{\mathbf{z}}^{jn}\right)^T\right)-\tilde{\mathbf{y}}^j\left(\tilde{\mathbf{y}}^j\right)^T \qquad (5)$$

Let,

$$\mathbf{C}^j \quad = \quad (1-\beta^{j0})\mathbf{S}^j - \tilde{\mathbf{P}}^j \tag{6}$$

Thus, using matrix inversion lemma and by definition of $\mathbf{J}^{j+} = (\mathbf{P}^{j+})^{-1}$ and $\mathbf{J}^{j-} = (\mathbf{P}^{j-})^{-1}$ we get,

$$\begin{aligned}\mathbf{J}^{j+} &= \mathbf{J}^{j-} + \mathbf{J}^{j-}\mathbf{K}^j \left(\left(\mathbf{C}^j\right)^{-1} - \mathbf{K}^{j^T}\mathbf{J}^{j-}\mathbf{K}^j \right)^{-1} \mathbf{K}^{j^T}\mathbf{J}^{j-} \\ &= \mathbf{J}^{j-} + \mathbf{G}^j \end{aligned} \tag{7}$$

where

$$\mathbf{G}^j \quad = \quad \mathbf{J}^{j-}\mathbf{K}^j \left(\left(\mathbf{C}^j\right)^{-1} - \mathbf{K}^{j^T}\mathbf{J}^{j-}\mathbf{K}^j \right)^{-1} \mathbf{K}^{j^T}\mathbf{J}^{j-}. \tag{8}$$

Thus, Equ. (3) and (7) are the JPDAF estimation equations in the information form. These results i.e., Eqns. (3,7,8,6) are shown as results in the main paper as Eqns. (18-21).

2. Comparison of algorithms in innovation form: Results

For simplicity, here we will drop the target superscript. Define the single step consensus operation $\mathcal{A}$ on a value a_i as,

$$\mathcal{A}(a_i) \quad = \quad a_i + \epsilon \sum_{i' \in \mathcal{N}_i} (a_{i'} - a_i) \tag{9}$$

Also,

$$\mathbf{B}_i \quad = \quad \sum_{i' \in \mathcal{N}_i \cup \{i\}} \mathbf{U}_{i'} \tag{10}$$

$$\mathbf{b}_i \quad = \quad \sum_{i' \in \mathcal{N}_i \cup \{i\}} \mathbf{u}_{i'} \tag{11}$$

Note that in general, $\mathbf{B}_i \neq N_C \mathcal{A}(\mathbf{U}_i)$ and $\mathbf{b}_i \neq N_C \mathcal{A}(\mathbf{u}_i)$. We can get the following results for a single consensus step, which will be derived later.

2.1. KCF

(Eqns. (4-5) in the main paper)

$$\hat{\mathbf{x}}_i^+ \quad = \quad \hat{\mathbf{x}}_i^- + \left(\mathbf{J}_i^- + \mathbf{B}_i\right)^{-1} \left(\mathbf{b}_i - \mathbf{B}_i \hat{\mathbf{x}}_i^-\right) + \frac{\epsilon}{1 + ||(\mathbf{J}_i^-)^{-1}||} (\mathbf{J}_i^-)^{-1} \sum_{i' \in \mathcal{N}_i} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right) \tag{12}$$

$$\mathbf{J}_i^+ \quad = \quad \mathbf{J}_i^- + \mathbf{B}_i \tag{13}$$

2.2. ICF

(Eqn. (40-41 in the main paper)

$$\hat{\mathbf{x}}_i^+ \quad = \quad \hat{\mathbf{x}}_i^- + \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \frac{1}{N_C} \mathbf{J}_{i'}^- \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right)\right) \tag{14}$$

$$\mathbf{J}_i^+ \quad = \quad N_C \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right) \tag{15}$$

2.3. MTIC

(Eqn. (42-43) in the main paper)

$$\hat{\mathbf{x}}_i^+ \quad = \quad \hat{\mathbf{x}}_i^- + \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \frac{\mathbf{J}_{i'}^-}{N_C} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right) + \mathcal{A}(\beta_{i0}\mathbf{U}_i\hat{\mathbf{x}}_i^-)\right) \tag{16}$$

$$\mathbf{J}_i^+ \quad = \quad N_C \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right) \tag{17}$$

3. Proofs of the results:

3.1. ICF

For a single consensus step, ICF estimate equations (Eqn. (9) in the main paper) can be written with the $\mathcal{A}()$ notation as,

$$\hat{\mathbf{x}}_i^+ = \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}\hat{\mathbf{x}}_i^-) + \mathcal{A}(\mathbf{u}_i)\right) \tag{18}$$

$$\mathbf{J}_i^+ = N_C \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right) \tag{19}$$

By adding and subtracting $\hat{\mathbf{x}}_i^-$ in RHS of (18) and denoting $\mathbf{F}_i = \frac{\mathbf{J}_i^-}{N_C}$, we get,

$$\hat{\mathbf{x}}_i^+ = \hat{\mathbf{x}}_i^- + (\mathcal{A}(\mathbf{F}_i) + \mathcal{A}(\mathbf{U}_i))^{-1} \left(\mathcal{A}(\mathbf{F}_i\hat{\mathbf{x}}_i^-) + \mathcal{A}(\mathbf{u}_i) - (\mathcal{A}(\mathbf{F}_i) + \mathcal{A}(\mathbf{U}_i))\,\hat{\mathbf{x}}_i^-\right) \tag{20}$$

$$= \hat{\mathbf{x}}_i^- + (\mathcal{A}(\mathbf{F}_i) + \mathcal{A}(\mathbf{U}_i))^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \mathcal{A}(\mathbf{F}_i\hat{\mathbf{x}}_i^-) - \mathcal{A}(\mathbf{F}_i)\hat{\mathbf{x}}_i^-\right) \tag{21}$$

Now,

$$\mathcal{A}(\mathbf{F}_i\hat{\mathbf{x}}_i^-) - \mathcal{A}(\mathbf{F}_i)x_i^- = \mathbf{F}_i\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \left(\mathbf{F}_i'\hat{\mathbf{x}}_{i'}^- - \mathbf{F}_i\hat{\mathbf{x}}_i^-\right) - \mathbf{F}_i\hat{\mathbf{x}}_i^- - \epsilon \sum_{i' \in \mathcal{N}_i} \left(\mathbf{F}_{i'}\hat{\mathbf{x}}_i^- - \mathbf{F}_i\hat{\mathbf{x}}_i^-\right) \tag{22}$$

$$= \epsilon \sum_{i' \in \mathcal{N}_i} \left(\mathbf{F}_{i'}\hat{\mathbf{x}}_{i'}^- - \mathbf{F}_i\hat{\mathbf{x}}_i^-\right) - \epsilon \sum_{i' \in \mathcal{N}_i} \left(\mathbf{F}_{i'}\hat{\mathbf{x}}_i^- - \mathbf{F}_i\hat{\mathbf{x}}_i^-\right) \tag{23}$$

$$= \epsilon \sum_{i' \in \mathcal{N}_i} \left(\mathbf{F}_{i'}\hat{\mathbf{x}}_{i'}^- - \mathbf{F}_i\hat{\mathbf{x}}_i^- - \mathbf{F}_{i'}\hat{\mathbf{x}}_i^- + \mathbf{F}_i\hat{\mathbf{x}}_i^-\right) \tag{24}$$

$$= \epsilon \sum_{i' \in \mathcal{N}_i} \mathbf{F}_{i'} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right) \tag{25}$$

Using this result in (21), we get,

$$\hat{\mathbf{x}}_i^+ = \hat{\mathbf{x}}_i^- + (\mathcal{A}(\mathbf{F}_i) + \mathcal{A}(\mathbf{U}_i))^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \mathbf{F}_{i'} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right)\right)$$

$$= \hat{\mathbf{x}}_i^- + \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \frac{\mathbf{J}_{i'}^-}{N_C} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right)\right) \tag{26}$$

Thus, we have the ICF equations in the innovation form (for a single consensus step), as (also showed as a result as Eqns. (40-41) in the main paper)

$$\hat{\mathbf{x}}_i^+ = \hat{\mathbf{x}}_i^- + \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \frac{\mathbf{J}_{i'}^-}{N_C} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right)\right) \tag{27}$$

$$\mathbf{J}_i^+ = N_C \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right) \tag{28}$$

3.2. MTIC

MTIC estimate equations (Eqns. (23-24) in the main paper) for a single consensus step, can be written with the $\mathcal{A}()$ notation as,

$$\hat{\mathbf{x}}_i^+ = \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\frac{\mathbf{J}_i}{N_C}\hat{\mathbf{x}}_i^-) + \mathcal{A}(\beta_0 \mathbf{U}_i \hat{\mathbf{x}}_i^-) + \mathcal{A}(\mathbf{u}_i)\right) \tag{29}$$

$$\mathbf{J}_i^+ = N_C \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{G}_i)\right) \tag{30}$$

By adding and subtracting $\hat{\mathbf{x}}_i^-$ in RHS of (29) and following the exact same steps of the proof in the previous section we can write the MTIC estimation equations for a single consensus step (also showed as a result as Eqns. (42-43) in the main paper),

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$$\hat{\mathbf{x}}_i^+ = \hat{\mathbf{x}}_i^- + \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{U}_i)\right)^{-1} \left(\mathcal{A}(\mathbf{u}_i) - \mathcal{A}(\mathbf{U}_i)\hat{\mathbf{x}}_i^- + \epsilon \sum_{i' \in \mathcal{N}_i} \frac{\mathbf{J}_{i'}^-}{N_C} \left(\hat{\mathbf{x}}_{i'}^- - \hat{\mathbf{x}}_i^-\right) + \mathcal{A}(\beta_{i0} \mathbf{U}_i \hat{\mathbf{x}}_i^-)\right) \tag{31}$$

$$\mathbf{J}_i^+ = N_C \left(\mathcal{A}(\frac{\mathbf{J}_i^-}{N_C}) + \mathcal{A}(\mathbf{G}_i)\right) \tag{32}$$
